MICRO WAVE ENGINEERING

(R20A0424)

LECTURE NOTES

B.TECH
(IV YEAR – I SEM)
(2023-24)

Prepared by:

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MALLA REDDY COLLEGE
OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

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MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

IV Year B.Tech. ECE- I Sem **L/T/P/C**
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(R20A0424) MICROWAVE ENGINEERING

COURSE OBJECTIVES:

1. To analyze Waveguides in Rectangular Coordinate Systems.
2. To Use S-parameter terminology to describe circuits.
3. To explain how microwave devices and circuits are characterized in terms of their "S" Parameters.
4. To Use microwave components such as isolators, Couplers, Circulators, Tees, Gyrators etc..
5. To give students an understanding of basic microwave devices (both amplifiers and oscillators).
6. To expose the students to the basic methods of microwave measurements.

UNIT I

Waveguides: Introduction, Microwave spectrum and bands, applications of Microwaves, Rectangular Waveguides-Solution of Wave Equation in Rectangular Coordinates, TE/TM mode analysis, Expressions for fields, Cutoff frequencies, dominant and degenerate modes, Mode characteristics - Phase and Group velocities, wavelengths and impedance relations, Impossibility of TEM Modes, Illustrative Problems.

UNIT II

Waveguide Components: Scattering Matrix - Significance, Formulation and properties, Wave guide multiport junctions - E plane and H plane Tees, Magic Tee, 2-hole Directional coupler, S Matrix calculations for E plane and H plane Tees, Magic Tee, Directional coupler, Ferrite components - Gyrator, Isolator, Circulator, Illustrative Problems.

UNIT III

Linear beam Tubes: Limitations and losses of conventional tubes at microwave frequencies, Classification of Microwave tubes, **O type tubes** - 2 cavity klystrons-structure, velocity modulation process and Applegate diagram, bunching process and small signal theory Expressions for o/p power and efficiency, Reflex Klystrons-structure, Velocity Modulation, Applegate diagram, power output, efficiency.

UNIT IV

Cross-field Tubes: Introduction, Magnetrons-different types, cylindrical travelling wave magnetron-Hull cutoff and Hartree conditions.

Microwave Semiconductor Devices: Introduction to Microwave semiconductor devices, classification, Transfer Electronic Devices, Gunn diode - principles, RWH theory, Characteristics, Basic modes of operation - Gunn oscillation modes, Introduction to Avalanche Transit time devices (brief treatment only), Illustrative Problems.

UNIT V

Microwave Measurements: Description of Microwave Bench – Different Blocks and their Features, Waveguide Attenuators – Resistive Card, Rotary Vane types; Microwave Power Measurement – Bolometer Method. Measurement of Attenuation, Frequency, VSWR, Impedance Measurements.

TEXT BOOKS:

1) Microwave Devices and Circuits – Samuel Y. Liao, PHI, 3rd Edition,1994.
2) Microwave and Radar Engineering- M.Kulkarni, Umesh Publications,1998.

REFERENCES:

1) Foundations for Microwave Engineering – R.E. Collin, IEEE Press, John Wiley, 2nd Edition, 2002.
2) Microwave Circuits and Passive Devices – M.L. Sisodia and G.S.Raghuvanshi, Wiley Eastern Ltd., New Age International Publishers Ltd., 1995.
3) Microwave Engineering Passive Circuits – Peter A. Rizzi, PHI, 1999.
4) Electronic and Radio Engineering – F.E. Terman, McGraw-Hill, 4th ed., 1955.
5) Elements of Microwave Engineering – R. Chatterjee, Affiliated East-West Press Pvt. Ltd., New Delhi,1988.

COURSE OUTCOMES

1) Understand the significance of microwaves and microwave transmission lines
2) Analyze the characteristics of microwave tubes and compare them
3) Be able to list and explain the various microwave solid state devices
4) Can set up a microwave bench for measuring microwave parameters
5) Expose to the basic methods of microwave measurements.

UNIT-I
Waveguides

Contents:

- Introduction

- Microwave spectrum and bands
- Applications of Microwaves
- Rectangular Waveguides-
- Solution of Wave Equation in Rectangular Coordinates
- TE/TM mode analysis- Expressions for fields
- Cutoff frequencies,
- Dominant and degenerate modes
- Mode characteristics - Phase and Group velocities, wavelengths and impedance relations, Impossibility of TEM Modes

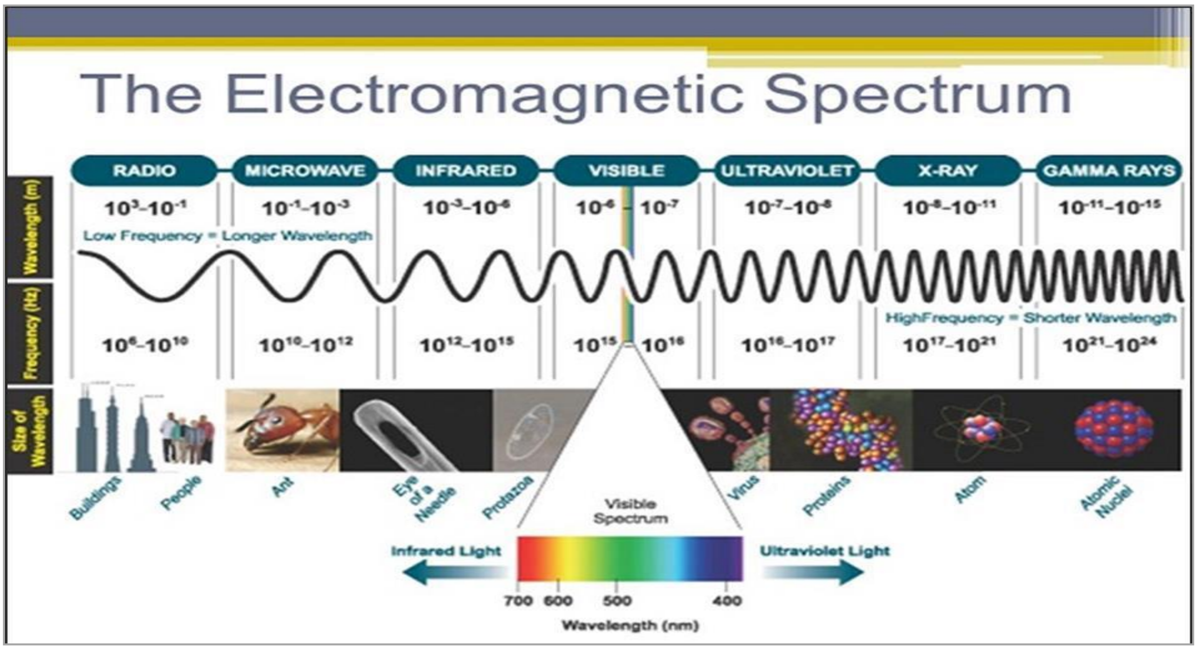
INTRODUCTION:

Microwaves are electromagnetic waves with frequencies between 300MHz (0.3GHz) and 300GHz in the electromagnetic spectrum. Radio waves are electromagnetic waves within the frequencies 30 KHz - 300GHz, and include microwaves. Microwaves are at the higher frequency end of the radio wave band and low frequency radio waves are at the lowerfrequency end. Mobile phones, phone mast antennas (base stations), DECT cordless phones, Wi-Fi,WLAN, WiMAX and Bluetooth have carrier wave frequencies within the microwave band of the electromagnetic spectrum, and are pulsed/modulated.Most Wi-Fi computers in schools use 2.45GHz (carrier wave), the same frequency as microwave ovens. Information about the frequencies can be found in Wi-Fi exposures and guidelines. It is worth noting that the electromagnetic spectrum is divided into different bands frequency. But the effects of electromagnetic radiation do not necessarily fit into these artificial divisions.

A waveguide consists of a hollow metallic tube of either rectangular or circular cross section used to guide electromagnetic wave. Rectangular waveguide is most commonly used as waveguide. Waveguides are used at frequencies in the microwave range. At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies. Hence microwave signals are propagated through the waveguides in order to minimize the losses.

Microwave Spectrum and bands:

Electromagnetic Spectrum consists of entire range of electromagnetic radiation. Radiationis the energy that travels and spreads out as it propagates. The type of electromagneticradiation that makes the electromagnetic spectrum is depicted in the following screenshot.



Frequency range	Wavelength	IEEE band
300KHz-3 MHz	1 km to 100 meters	MF
3-30 MHz	100 meters to 10 meters	HF
30-300 MHz	10 meters to 1 meter	VHF
300 MHz -3 GHz*	1 meter to 10 cm	UHF
1-2 GHz	30 cm to 15 cm	**L** band
2-4 GHz	15 cm to 5 cm	**S** band
4-8 GHz	5 cm to 3.75 cm	**C** band
8-12 GHz	3.75 cm to 2.5 cm	**X** band
12-18 GHz	2.5 cm to 1.6 cm	K_u band
18-26 GHz	1.6 cm to 1.2 cm	**K** band
26-40 GHz	1.6 cm to 750 mm	K_a band
40-75 GHz	750 mm to 40 mm	**V** band
75 to 111 GHz	40 mm to 28mm	**W** band
Above 111 GHz	"millimeter wave"	

PropertiesofMicrowaves

Following are the main properties of Microwaves.

- Microwaves are the waves that radiate electromagnetic energy with shorter wavelength.
- Microwaves are not reflected by Ionosphere.
- Microwaves travel in a straight line and are reflected by the conducting surfaces.
- Microwaves are easily attenuated within shorter distances.
- Microwave currents can flow through a thin layer of a cable.

AdvantagesofMicrowaves

There are many advantages of Microwaves such as the following:

- Supports larger bandwidth and hence more information is transmitted. For this reason, microwaves are used for point-to-point communications.
- More antenna gain is possible.
- Higher data rates are transmitted as the bandwidth is more.
- Antenna size gets reduced, as the frequencies are higher.
- Low power consumption as the signals are of higher frequencies.
- Effect of fading gets reduced by using line of sight propagation.
- Provides effective reflection area in the radar systems.
- Satellite and terrestrial communications with high capacities are possible.
- Low-cost miniature microwave components can be developed.
- Effective spectrum usage with wide variety of applications in all available frequency ranges of operation.

DisadvantagesofMicrowaves

There are a few disadvantages of Microwaves such as the following:

- Cost of equipment or installation cost is high.
- They are hefty and occupy more space.
- Electromagnetic interference may occur.
- Variations in dielectric properties with temperatures may occur.
- Inherent inefficiency of electric power.

ApplicationsofMicrowaves

There are a wide variety of applications for Microwaves, which are not possible for other radiations. They are -

Wireless Communications

- For long distance telephone calls
- Bluetooth
- WIMAX operations
- Outdoor broadcasting transmissions
- Broadcast auxiliary services
- Direct Broadcast Satellite (DBS)
- Personal Communication Systems (PCSs)
- Wireless Local Area Networks (WLANs)
- Cellular Video (CV) systems
- Automobile collision avoidance system

Electronics

- Fast jitter-free switches
- Phase shifters
- HF generation
- Tuning elements
- ECM/ECCM (Electronic Counter Measure) systems
- Spread spectrum systems

Commercial Uses

- Burglar alarms
- Garage door openers
- Police speed detectors
- Identification by non-contact methods
- Cell phones, pagers, wireless LANs
- Satellite television, XM radio
- Motion detectors
- Remote sensing

Navigation

- Global navigation satellite systems
- Global Positioning System (GPS)

Military and Radar

- Radars to detect the range and speed of the target.
- SONAR applications
- Air traffic control
- Weather forecasting
- Navigation of ships
- Minesweeping applications
- Speed limit enforcement
- Military uses microwave frequencies for communications and for the above mentioned applications.

Research Applications

- Atomic resonances
- Nuclear resonances

Radio Astronomy

- Mark cosmic microwave background radiation
- Detection of powerful waves in the universe
- Detection of many radiations in the universe and earth's atmosphere

Food Industry

- Microwave ovens used for reheating and cooking
- Food processing applications
- Pre-heating applications
- Pre-cooking
- Roasting food grains/beans
- Drying potato chips
- Moisture levelling
- Absorbing water molecules

Industrial Uses

- Vulcanizing rubber
- Analytical chemistry applications
- Drying and reaction processes
- Processing ceramics
- Polymer matrix
- Surface modification
- Chemical vapor processing
- Powder processing
- Sterilizing pharmaceuticals
- Chemical synthesis
- Waste remediation
- Power transmission
- Tunnel boring
- Breaking rock/concrete
- Breaking up coal seams
- Active denial systems

Semiconductor Processing Techniques

- Reactive ion etching

- Chemical vapor deposition

Spectroscopy

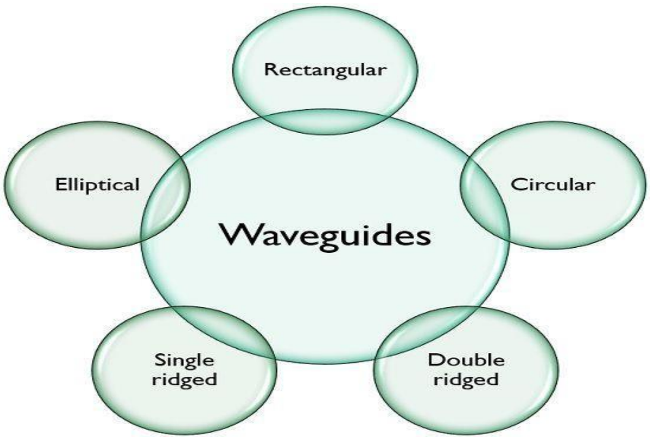
- Electron Paramagnetic Resonance (EPR or ESR) Spectroscopy
- To know about unpaired electrons in chemicals
- To know the free radicals in materials
- Electron chemistry

Medical Applications

- Monitoring heartbeat
- Lung water detection
- Tumor detection
- Regional hyperthermia
- Therapeutic applications
- Local heating
- Angioplasty
- Microwave tomography
- Microwave Acoustic imaging

Types of waveguides

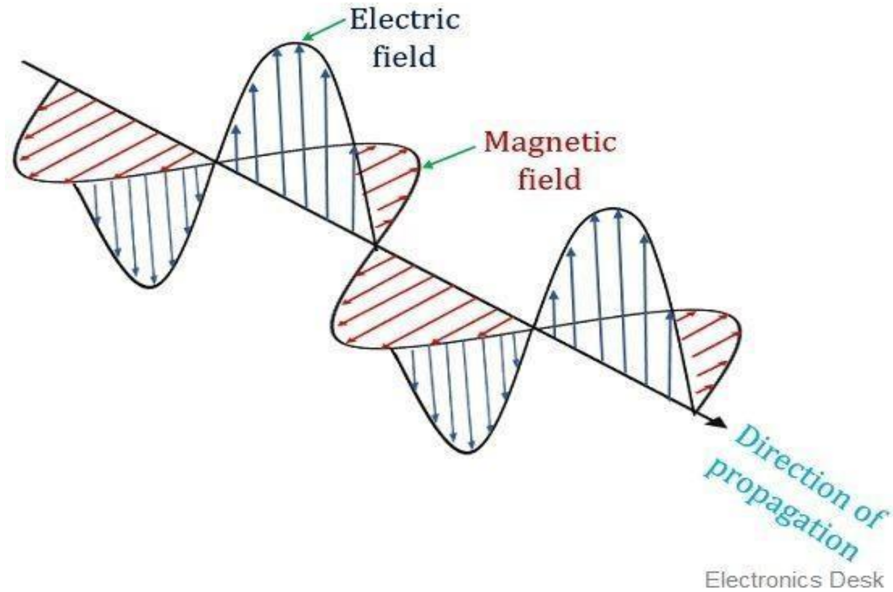
Waveguides are majorly classified as rectangular or circular but these are basically of 5 different types:



Modes of propagation in a Waveguide

When an electromagnetic wave is transmitted through a waveguide. Then it has two field components that oscillate mutually perpendicular to each other. Out of the two one is electric field and the other is a magnetic field.

The figure below represents the propagation of an electromagnetic wave in the z-direction with the two field components:



The propagation of wave inside the waveguide originates basically 2 modes. However, overall basically 3 modes exist, which are as follows:

- **Transverse Electric wave**:
  In this mode of wave propagation, the electric field component is totally transverse to the direction of wave propagation whereas the magnetic field is not totally transverse to the direction of wave propagation. It is abbreviated as TE mode.

$$E_z = 0; H_z \neq 0$$

- **Transverse Magnetic wave**:
  In this mode of wave propagation, the magnetic field component is totally transverse to the direction of wave propagation while the electric field is not totally transverse to the direction of wave propagation. It is abbreviated as TM mode.

$$E_z \neq 0; H_z = 0$$

- **Transverse electromagnetic wave**:
  In this mode of wave propagation, both the field components i.e., electric and magnetic fields are totally transverse to the direction of wave propagation. It is abbreviated as TEM mode.

$$E_z = H_z = 0$$

It is to be noted here that, TEM mode is not supported in waveguides. As for the TEM mode, there is a need for the presence of two conductors and we already know that a waveguide is a single hollow conductor.

Parameters of a Waveguide:

- **Cut-off wavelength**: It the maximum signal wavelength of the transmitted signal that can be propagated within the waveguide without any attenuation. This means up to cut-off wavelength, a microwave signal can be easily transmitted through the waveguide. It is denoted by λ_c.
- **Group velocity**: Group velocity is the velocity with which wave propagates inside the waveguide. If the transmitted carrier is modulated, then the velocity of the modulation envelope is somewhat less as compared to the carrier signal. This velocity of the envelope is termed as group velocity. It is represented by V_g.
- **Phase velocity**: It is the velocity with which the transmitted wave changes its phase during propagation. Or we can say it is basically the velocity of a particular phase of the propagating wave. It is denoted by V_p.
- **Wave Impedance**: It is also known as the characteristic impedance. It is defined as the ratio of the transverse electric field to that of the transverse magnetic field during wave propagation at any point inside the waveguide. It is denoted by Z_g.

Advantages of waveguides

1. In waveguides, the power loss during propagation is almost negligible.
2. Waveguides have the ability to manage large-signal power.
3. As waveguides possess a simple structure thus their installation is somewhat easy.

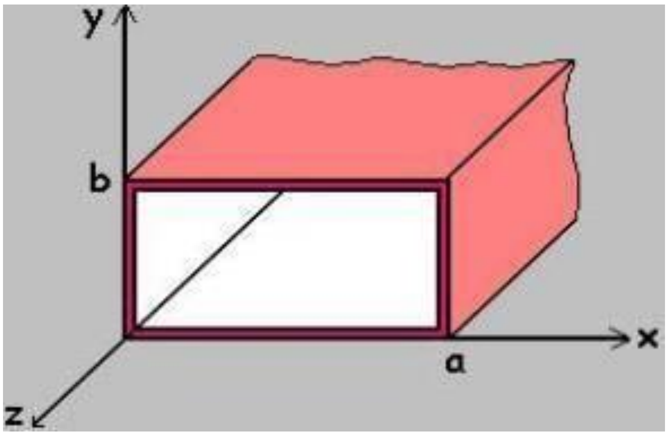
Disadvantages of waveguides

1. Its installation and manufacturing cost is high.
2. Waveguides are generally rigid in nature and hence sometimes causes difficulty in applications where tube flexibility is required.
3. It is somewhat large in size and bulkier as compared to other transmission lines.
   It is noteworthy in the case of waveguides that their diameter must have some certain value in order to have proper signal propagation. This is so because if its diameter is very small and the wavelength of the signal to be propagated is large (or signal frequency is small) then it will not be propagated properly.

   So, the signal frequency must be greater than the cutoff frequency in order to have a proper signal transmission.

Wave propagation in rectangular waveguide:

Consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along x-axis, width along y axis and the wave assumed to propagate along the z-direction. Waveguide is filled with air and in that no TEM wave is exist.



We assumed that wave direction is along Z-direction then the wave equations are

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave } (E_z = 0) \rightarrow (1)$$

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{for TM wave } (Hz = 0) \rightarrow (2)$$

$$\nabla^2 \mathbf{E}_z = -\omega^2 \mu \varepsilon \mathbf{E}_z$$

Expanding $\nabla^2 \mathbf{E}_z$ in rectangular co-ordinate system

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + \frac{\partial^2 \mathbf{E}_z}{\partial z^2} = -\omega^2 \mu \varepsilon \mathbf{E}_z \quad (7)$$

Since the wave is propagating in the 'z' direction we have the operator.

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

Substituting this operator in Eq. 4.23, we get

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + \gamma^2 \mathbf{E}_z = -\omega^2 \mu \varepsilon \mathbf{E}_z$$

or

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon)\, \mathbf{E}_z = 0 \quad (8)$$

Let $\gamma^2 + \omega^2 \mu \varepsilon = h^2$, be a constant, then Eq. (8) can be rewritten as

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + h^2 \mathbf{E}_z = 0 \quad \text{for TM wave} \quad (9)$$

$$\frac{\partial^2 \mathbf{H}_z}{\partial x^2} + \frac{\partial^2 \mathbf{H}_z}{\partial y^2} + h^2 \mathbf{H}_z = 0 \quad \text{for TM wave} \quad (10)$$

By solving the above partial differential equations, we get solutions for E_z and H_z. Using Maxwell's equation, it is possible to find the various components along x and y directions [$\mathbf{E}_x$, $\mathbf{H}_x$, $\mathbf{E}_y$, $\mathbf{H}_y$].

From Maxwell's 1st equation, we have

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

Expanding $\nabla \times \mathbf{H}$,

i.e.,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{H}_x & \mathbf{H}_y & \mathbf{H}_z \end{vmatrix} = j\omega\varepsilon\ [\hat{i}\,\mathbf{E}_x + \hat{j}\,\mathbf{E}_y + \hat{k}\,\mathbf{E}_z]$$

Replacing $\frac{\partial}{\partial z} = -\gamma$ (an operator), we get,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ \mathbf{H}_x & \mathbf{H}_y & \mathbf{H}_z \end{vmatrix} = j\omega\varepsilon\ [\hat{i}\,\mathbf{E}_x + \hat{j}\,\mathbf{E}_y + \hat{k}\,\mathbf{E}_z]$$

Equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ (after expanding) we get,

$$\frac{\partial \mathbf{H}_z}{\partial y} + \gamma\,\mathbf{H}_y = j\omega\varepsilon\mathbf{E}_x \quad \text{———} \quad (11)$$

$$\frac{\partial \mathbf{H}_z}{\partial x} + \gamma\,\mathbf{H}_x = -j\omega\varepsilon\mathbf{E}_y \quad \text{———} \quad (12)$$

$$\frac{\partial \mathbf{H}_y}{\partial x} - \frac{\partial\,\mathbf{H}_x}{\partial y} = j\omega\varepsilon\mathbf{E}_z \quad \text{———} \quad (13)$$

Similarly from Maxwell's 2nd equation, we have

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

Expanding $\nabla \times \mathbf{E}$, we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{E}_x & \mathbf{E}_y & \mathbf{E}_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ \mathbf{E}_x & \mathbf{E}_y & \mathbf{E}_z \end{vmatrix} = -j\omega\mu\ [\hat{i}\,\mathbf{H}_x + \hat{j}\,\mathbf{H}_y + \hat{k}\,\mathbf{H}_z]$$

Expanding and equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$, we get

$$\frac{\partial \mathbf{E}_z}{\partial y} + \gamma\,\mathbf{E}_y = -j\omega\mu\mathbf{H}_x \quad \text{———} \quad (14)$$

$$\frac{\partial \mathbf{E}_z}{\partial x} + \gamma \mathbf{E}_x = + j\omega\mu \mathbf{H}_y \qquad (15)$$

$$\frac{\partial \mathbf{E}_y}{\partial x} - \frac{\partial \mathbf{E}_x}{\partial y} = - j\omega\mu \mathbf{H}_z \qquad (16)$$

From eq..15 write Hy and substitute in eq..11 to find Ex

$$\mathbf{H}_y = \frac{1}{j\omega\mu}\frac{\partial \mathbf{E}_z}{\partial x} + \frac{\gamma}{j\omega\mu}\mathbf{E}_x$$

Substituting for Hy in Eq. 4.28, we get,

$$\frac{\partial \mathbf{H}_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial \mathbf{E}_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}\mathbf{E}_x = j\omega\varepsilon\mathbf{E}_x$$

or

$$\mathbf{E}_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu}\frac{\partial \mathbf{E}_z}{\partial x} + \frac{\partial \mathbf{H}_z}{\partial y}$$

Multiplying by $j\omega\mu$, we get

$$\mathbf{E}_x\,[-\omega^2\mu\varepsilon - \gamma^2] = \gamma\frac{\partial \mathbf{E}_z}{\partial x} + j\omega\mu\frac{\partial \mathbf{H}_z}{\partial y}$$

or

$$\mathbf{E}_x\,[-(\gamma^2 + \omega^2\mu\varepsilon)] = \gamma\frac{\partial \mathbf{E}_z}{\partial x} + j\omega\mu\frac{\partial \mathbf{H}_z}{\partial y}$$

where $\gamma^2 + \omega^2\mu\varepsilon = h^2$

Dividing by $-h^2$ throughout, we get

$$\mathbf{E}_x = \frac{-\gamma}{h^2}\frac{\partial \mathbf{E}_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial \mathbf{H}_z}{\partial y} \qquad (17)$$

Similarly
$$\mathbf{E}_y = \frac{-\gamma}{h^2}\frac{\partial \mathbf{E}_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial \mathbf{H}_z}{\partial x} \qquad (18)$$

and
$$\mathbf{H}_x = \frac{-\gamma}{h^2}\frac{\partial \mathbf{H}_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial \mathbf{E}_z}{\partial y} \qquad (19)$$

and
$$\mathbf{H}_y = \frac{-\gamma}{h^2}\frac{\partial \mathbf{H}_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial \mathbf{E}_z}{\partial x} \qquad (20)$$

These, equations give a general relationship for field components within a waveguide.

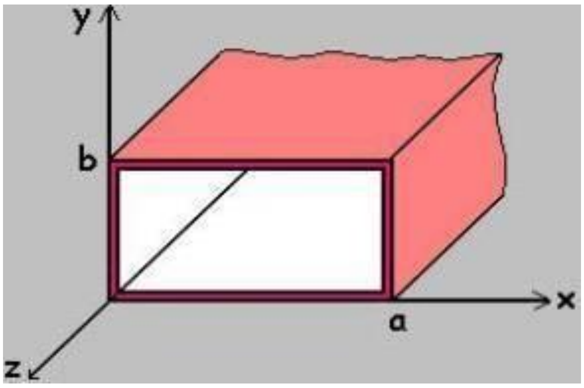
Rectangular Wave guides:

Rectangular waveguides are the one of the earliest type of the transmission lines. They are used in many applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available for various standard waveguide bands between 1 GHz to above 220 GHz.

A rectangular waveguide supports TM and TE modes but not TEM waves because we cannot define a unique voltage since there is only one conductor in a rectangular waveguide. The shape of a rectangular waveguide is as shown below. A material with permittivity e and permeability m fills the inside of the conductor.

A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the *cut- off frequency*.

Here, we will discuss TM mode rectangular waveguides and TE mode rectangular waveguides separately. Let's start with the TM mode



TM Modes

Consider the shape of the rectangular waveguide above with dimensions a and b (assume a>b) and the parameters **e** and **m.** For TM waves $H_z = 0$ and E_z should be solved from equation for TM mode;

Ñ$^2_{xy}$ $E_z^0 + h^2 E_z^0 = 0$

Since $E_z(x,y,z) = E_z^0(x,y)e^{-gz}$, we get the following equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)E_z^0(x,y) = 0$$

If we use the method of separation of variables, that is $E_z^0(x,y)=X(x).Y(y)$ we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as k_x^2, we get;

$$\frac{d^2X(x)}{dx^2}+k_x^2X(x)=0$$

$$\frac{d^2Y(y)}{dy^2}+k_y^2Y(y)=0$$

where $k_y^2=h^2-k_x^2$

Now, we should solve for X and Y from the preceding equations. Also we have the boundary conditions of;

$E_z^0(0,y)=0$

$E_z^0(a,y)=0$

$E_z^0(x,0)=0$

$E_z^0(x,b)=0$

From all these, we conclude that

X(x) is in the form of **sin k_xx**, where k_x=mp/a, m=1,2,3,…

Y(y) is in the form of **sin k_yy**, where k_y=np/b, n=1,2,3,…

So the solution for $E_z^0(x,y)$ is

$$E_z^0(x,y)=E_0\ \mathbf{sin}\left(\frac{m\pi}{a}x\right)\mathbf{sin}\left(\frac{n\pi}{b}y\right)\ \text{(V/m)}$$

From $k_y^2=h^2-k_x^2$, we have;

$$\boxed{h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

For TM waves, we have

$$H_x^0=\frac{jw\varepsilon}{h^2}\frac{\partial E_z^0}{\partial y}$$

$$H_y^0=-\frac{jw\varepsilon}{h^2}\frac{\partial E_z^0}{\partial x}$$

$$E_x^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x}$$

$$E_y^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y}$$

From these equations, we get

$$E_x^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$\boxed{E_y^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_x^0(x,y) = \frac{jw\varepsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_y^0(x,y) = -\frac{jw\varepsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)}$$

where

$$\boxed{\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Here, m and n represent possible modes and it is designated as the TM_{mn} mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

When we observe the above equations we see that *for TM modes in rectangular waveguides, neither m nor n can be zero*. This is because of the fact that the field expressions are identically zero if either m or n is zero. Therefore, the lowest mode for rectangular waveguide TM mode is TM_{11} .

Here, the cut-off wave number is

$$\boxed{k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

and therefore,

$$\boxed{\beta = \sqrt{k^2 - k_c^2}}$$

The cut-off frequency is at the point where **g** vanishes. Therefore,

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\ (Hz)$$

Since *l=u/f*, we have the cut-off wavelength,

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\ (m)}$$

At a given operating frequency f, only those frequencies, which have $f_c<f$ will propagate. The modes with $f<f_c$ will lead to an imaginary **b** which means that the field components will decay exponentially and will not propagate. Such modes are called *cut-off* or *evanescent* modes.

The mode with the lowest cut-off frequency is called the *dominant mode*. Since TM modes for rectangular waveguides start from TM_{11} mode, the dominant frequency is

$$(f_c)_{11} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}\ (Hz)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for E_x and H_y (see the equations above);

$$Z_{TM} = \frac{E_x}{H_y} = \frac{\gamma}{jw\varepsilon} = \frac{j\beta}{jw\varepsilon} \Rightarrow Z_{TM} = \frac{\beta\eta}{k}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g = \frac{2\pi}{\beta}} > \frac{2\pi}{k} = \lambda$$

Which is thus greater than **l**, the wavelength of a plane wave in the filling medium.

The phase velocity is

$$u_p = \frac{w}{\beta} > \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Which is greater than the speed of light (plane wave) in the filling material.

Attenuation for propagating modes results when there are losses in the dielectric and in the imperfectly conducting guide walls. The attenuation constant due to the losses in the dielectric can be found as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

TE Modes

Consider again the rectangular waveguide below with dimensions a and b (assume a>b) and the parameters **e** and **m.**

For TE waves $E_z = 0$ and H_z should be solved from equation for TE mode;

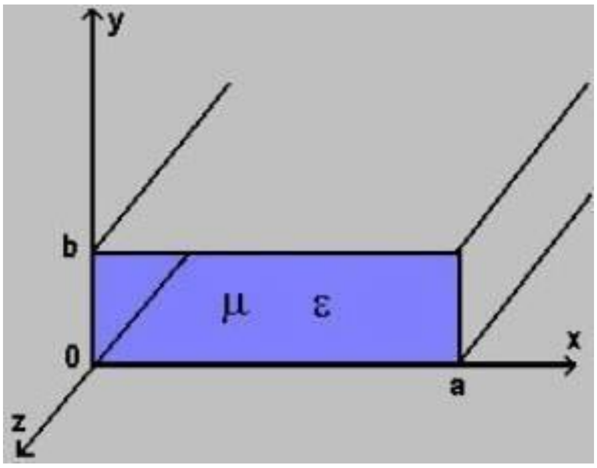
Ñ$^2_{xy}$ H_z + h^2 H_z = 0

Since $H_z(x,y,z) = H_z^0(x,y)e^{-gz}$, we get the following equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)H_z^0(x,y) = 0$$

If we use the method of separation of variables, that is $H_z^0(x,y)=X(x).Y(y)$ we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2$$



Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as $k_x^{\,2}$, we get;

$$\frac{d^2X(x)}{dx^2} + k_x^2X(x) = 0$$

$$\frac{d^2Y(y)}{dy^2} + k_y^2Y(y) = 0$$

where $k_y^2=h^2-k_x^2$

Here, we must solve for X and Y from the preceding equations. Also we have the following boundary conditions:

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \quad \text{at } x=0$$

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \quad \text{at } x=a$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \quad \text{at } y=0$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \quad \text{at } y=b$$

From all these, we get

$$H_z^0(x,y)=H_0\ \mathbf{cos}\left(\frac{m\pi}{a}x\right)\mathbf{cos}\left(\frac{n\pi}{b}y\right) \quad \text{(A/m)}$$

From $k_y^2=h^2-k_x^2$, we have;

$$\boxed{h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

For TE waves, we have

$$H_x^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}$$

$$H_y^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_x^0=-\frac{j w\mu}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_y^0=-\frac{j w\mu}{h^2}\frac{\partial H_z^0}{\partial x}$$

From these equations, we obtain

$$E_x^0(x,y) = \frac{jw\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$\boxed{E_y^0(x,y) = -\frac{jw\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_x^0(x,y) = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_y^0(x,y) = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)}$$

where

$$\boxed{\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

As explained before, m and n represent possible modes and it is shown as the TE_{mn} mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

Here, the cut-off wave number is

$$\boxed{k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

and therefore,

$$\beta = \sqrt{k^2 - k_c^2}$$

The cut-off frequency is at the point where **g** vanishes. Therefore,

$$\boxed{f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\ (Hz)}$$

Since *l=u/f*, we have the cut-off wavelength,

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\ (m)}$$

At a given operating frequency f, only those frequencies, which have $f>f_c$ will propagate. The modes with $f<f_c$ will not propagate.

The mode with the lowest cut-off frequency is called the *dominant mode*. Since TE_{10} mode is the minimum possible mode that gives nonzero field expressions for rectangular waveguides, it is the dominant mode of a rectangular waveguide with a>b and so the dominant frequency is

$$(f_c)_{10} = \frac{1}{2a\sqrt{\mu\varepsilon}} (Hz)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for E_x and H_y (see the equations above);

$$Z_{TE} = \frac{E_x}{H_y} = \frac{jw\mu}{\gamma} = \frac{jw\mu}{j\beta} \Rightarrow Z_{TE} = \frac{k\eta}{\beta}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g = \frac{2\pi}{\beta}} > \frac{2\pi}{k} = \lambda$$

Which is thus greater than **l**, the wavelength of a plane wave in the filling medium.

The phase velocity is

$$u_p = \frac{w}{\beta} > \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Which is greater than the speed of the plane wave in the filling material.

The attenuation constant due to the losses in the dielectric is obtained as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

After some manipulation, we get

$$\alpha_d = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{k^2 \tan\delta}{2\beta}$$

Example:

Consier a length of air-filled copper X-band waveguide, with dimensions a=2.286cm, b=1.016cm. Find the cut-off frequencies of the first four propagating modes.

Solution:

From the formula for the cut-off frequency

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \xrightarrow{\text{air-filled}} \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\ (Hz)$$

Rectangular wave guide Cutoff frequencies:

The cut-off frequency is the frequency above which the waveguide offers minimum attenuation to the propagation of the signal.

Frequencies below the cut-off frequency are attenuated by the waveguide.

The dominant mode in a waveguide is the propagation mode with the lowest cut-off frequency.

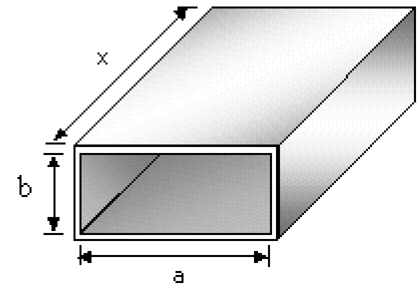
Waveguides are hollow metallic structures that carry signals from one end to another. All the signals that propagate through a waveguide are above a certain frequency, called the cut-off frequency. Below the cut- off frequency, waveguides fail to transfer wave energy or propagate waves.

Cut-off frequency can also be described as the frequency above which the waveguide offers minimum attenuation to the propagation of the signal. Frequencies below the cut-off frequency are attenuated by the waveguide. The signal propagation through a waveguide is dependent on the signal wavelength as well. When a wavelength is too long, the waveguide stops carrying signals and becomes inoperative.

Consider a rectangular waveguide with width ‘a’ and thickness ‘b’. Let TE_{mn} be the mode active in the waveguide. To calculate the cut-off frequency f_c of the rectangular waveguide, use the following equation, where c is the speed of the light inside the waveguide and m and n are the numbers that define the mode of propagation.

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \quad (1)$$

The lower cutoff frequency (or wavelength) for a particular mode in rectangular waveguide is determined by the following equations (note that the length, x, has no bearing on the cutoff frequency):



$$(f_c)_{mn} = \frac{1}{2 \cdot \pi \cdot \sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m \cdot \pi}{a}\right)^2 + \left(\frac{n \cdot \pi}{b}\right)^2} \quad [Hz]$$

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \quad [m]$$

where	a =	Inside width (m), longest dimension
	b =	Inside height (m), shortest dimension
	m =	Number of ½-wavelength variations of fields in the "a" direction
	n =	Number of ½-wavelength variations of fields in the "b" direction
	ε =	Permittivity (8.854187817E-12 for free space)
	µ =	Permeability (4πE-7 for free space)

Dominant and Degenerate Modes:

Dominant Mode

The dominant mode in a waveguide is the propagation mode with the lowest cut-off frequency. The criterion for wave propagation through the waveguide is that the operating frequency should be greater than the dominant mode cut-off frequency. There will be minimum degradation of the signal in the dominant mode.

Degenerate Mode:

We know that the rectangular waveguide does not support TEM mode. It allows either TE mode or TM mode. If any two modes of propagation share the same cut-off frequency, such modes are called degenerate modes. The modes TE_{mn} and TM_{mn} are degenerate modes in a rectangular waveguide.
The rectangular waveguide cut-off frequency is a critical specification associated with rectangular waveguides, below which there is no signal propagation.

Wavelength in a waveguide It is known from Plane Wave tutorial that the wavelength of a plane wave is strictly related to the wave frequency. The wavelength in a waveguide is considered as a wavelength in a direction of wave propagation and its dependence on wave frequency is defined as follows:

$$\lambda_z = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

λ0 is a wavelength in a free space at a given frequency and λc stands for the cutoff wavelength for a given waveguide dimensions and waveguide mode.

UNIT-II

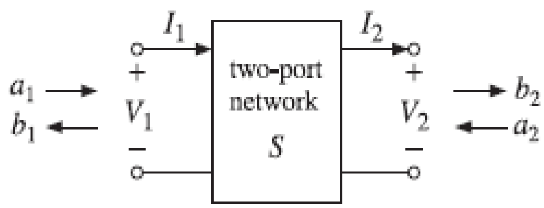
WAVEGUIDE COMPONENTS

Contents:

- Scattering Matrix - Significance, Formulation and properties
- Wave guide multiport junctions -E plane and H plane Tees, Magic Tee
- 2-hole Directional coupler
- S Matrix calculations for E plane and H plane Tees, Magic Tee, Directional coupler,
- Ferrite components - Gyrator, Isolator, Circulator, Illustrative Problems.

SCATTERING PARAMETERS

- Linear two-port (and multi-port) networks are characterized by a number of equivalent circuit parameters, such as their transfer matrix, impedance matrix, admittance matrix, and scattering matrix. Fig. shows a typical two-port network.



- The transfer matrix, also known as the ABCD matrix, relates the voltage and current at port 1 to those at port 2, whereas the impedance matrix relates the two voltages V_1, V_2 to the two currents I_1, I_2.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \text{(transfer matrix)}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} \qquad \text{(impedance matrix)}$$

- Thus, the transfer and impedance matrices are the 2×2 matrices:
- The admittance matrix is simply the inverse of the impedance matrix, $Y = Z^{-1}$. The scattering matrix relates the outgoing waves b1, b2 to the incoming waves a1, a2 that are incident on the two-port:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \text{(scattering matrix)}$$

- The matrix elements S11, S12, S21, S22 are referred to as the scattering parameters or the S- parameters. The parameters S11, S22 have the meaning of reflection coefficients, and S21, S12, themeaning of transmission coefficients.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \text{(transfer matrix)}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} \qquad \text{(impedance matrix)}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad \text{(scattering matrix)}$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

S- THE SCATTERING MATRIX

- The scattering matrix is defined as the relationship between the forward and backward moving waves. For a two-port network, like any other set of two-port parameters, the scattering matrix is a 2| matrix.

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

PROPERTIES OF SMATRIX:

In general the scattering parameters are complex quantities having the following Properties:

Property (1)

- When any Z port is perfectly matched to the junction, then there are no reflections from that S = 0. If all the ports are perfectly matched, then the leading diagonal II elements will all be zero.

Property (2)

- Symmetric Property of S-matrix: If a microwave junction satisfies reciprocity condition and if there are no active devices, then S parameters are equal to their corresponding transposes.

i.e., $\mathbf{S_{ij}=S_{ji}}$

Property (3)

- Unitary property for a lossless junction - This property states that for any lossless network, the sum of the products of each term of anyone row or anyone column of the [SJ matrix with its complex conjugate is unity

Property (4)

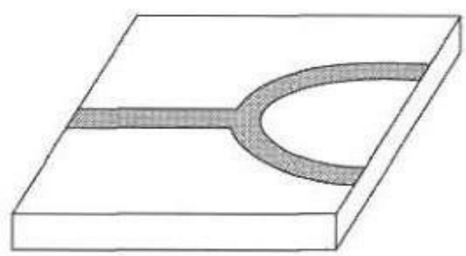
Phase - Shift Property:

- Complex S-parameters of a network are defined with respect to the positions of the port or reference planes. For a two-port network with unprimed reference planes 1 and 2 as shown in figure 4.6, the S- parameters have definite values.

WAVEGUIDE MULTIPORT JUNCTIONS:

T-JUNCTION POWER DIVIDER USING WAVEGUIDE:

- The T-junction power divider is a 3-port network that can be constructed either from a transmission line or from the waveguide depending upon the frequency of operation.



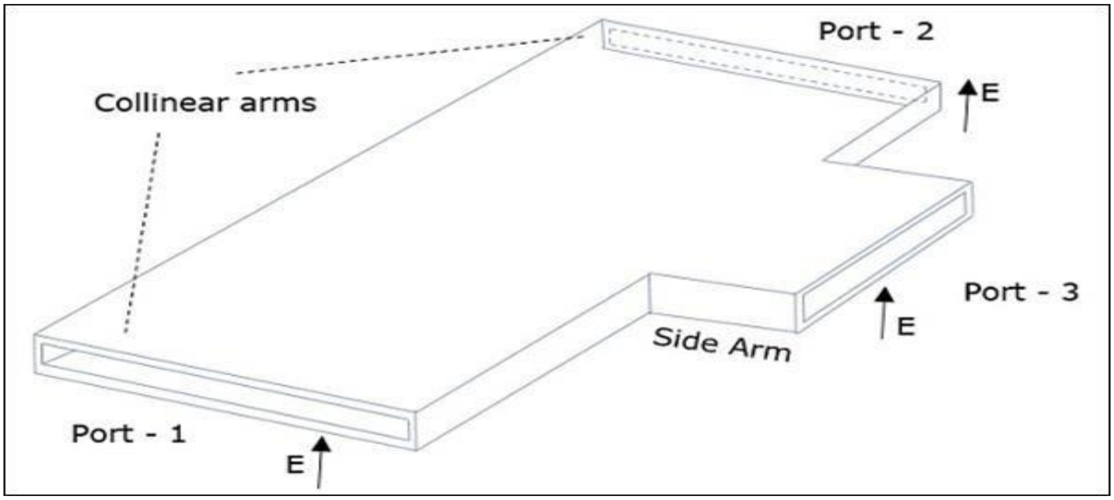
For very high frequency, power divider using waveguide is of 4 types

- H-Plane Tee
- E-Plane Tee
- E-H Plane Tee/Magic Tee
- Rat Race Tee

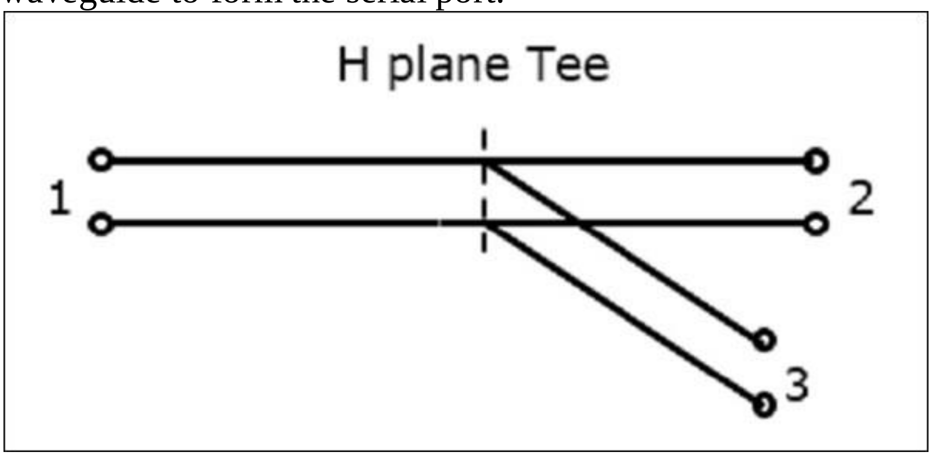
H-Plane Tee

An H-Plane Tee junction is designed by bestowing a simple waveguide to a rectangular waveguide which previously has two ports. The arms of rectangular waveguides make two ports called collinear ports i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or H-arm. This H-plane Tee is also called as Shunt Tee.

As the axis of the side arm is similar to the magnetic field, this junction is called H-Plane Tee junction. This is also called as Current junction, as the magnetic field splits itself into arms. The cross-sectional details of H-plane tee can be agreed by the resulting figure.



The following figure shows the connection made by the sidearm to the bi-directional waveguide to form the serial port.



Properties of H-Plane Tee

The properties of H-Plane Tee can be defined by its $[S]_{3\times3}$matrix.

1. It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{........ Equation 1}$$

2. Scattering coefficients S_{13} and S_{23} are equal here as the junction is symmetrical in plane

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} \;\; S_{23} = S_{32} = S_{13} \;\; S_{13} = S_{31}$$

3. The port is perfectly matched to the junction.

$$S_{33} = 0$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$ **........ Equation 2**

4. We can say that we have four unknowns, considering the symmetry property.

$$[S][S]* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From the Unitary property

$$R_1C_1 : S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$ **........ Equation 3**

$$R_2C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$ **......... Equation 4**

$$R_3C_3 : |S_{13}|^2 + |S_{13}|^2 = 1$$ **......... Equation 5**

$$R_3C_1 : S_{13}S_{11}^* - S_{13}S_{12}^* = 0$$ **......... Equation 6**

$$2|S_{13}|^2 = 1 \quad or \quad S_{13} = \frac{1}{\sqrt{2}}$$ **......... Equation 7**

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22}$$ **......... Equation 8**

From the Equation 6,

$$S_{13}(S_{11}^* + S_{12}^*) = 0$$

$$S_{13} \neq 0, S_{11}^{*} + S_{12}^{*} = 0, \; or \; S_{11}^{*} = -S_{12}^{*}$$

$$S_{11} = -S_{12} \; or \; S_{12} = -S_{11}$$ .... **Equation 9**

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1 \quad or \quad 2|S_{11}|^2 = \frac{1}{2} \quad or \quad S_{11} = \frac{1}{2}$$ ..... **Equation 10**

From equation 8 and 9,

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

We know that [b] = [s][a]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

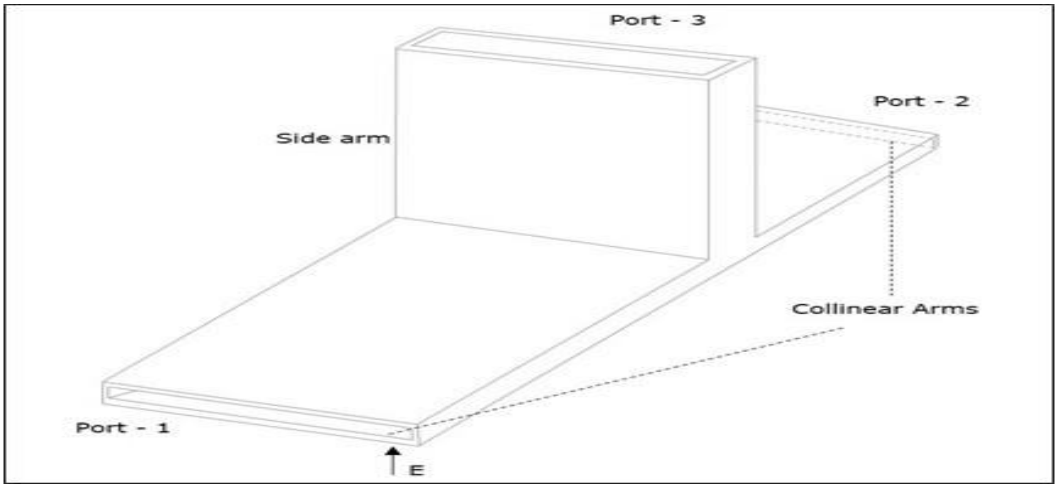
This is the scattering matrix for H-Plane Tee, which explains its scattering properties.

E-Plane Tee

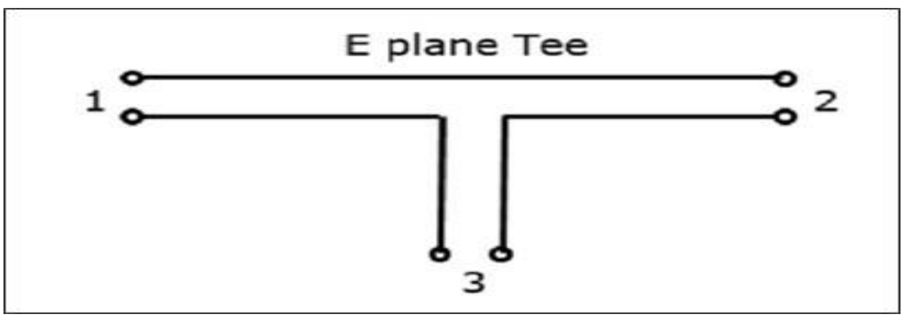
An E-Plane Tee junction is formed by attaching a simple waveguide to the broader dimension of a rectangular waveguide, which already has two ports. The arms of rectangular waveguides make two ports called **collinear ports** i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or **E-arm**. T his E-plane Tee is also called as **Series Tee**.

As the axis of the side arm is parallel to the electric field, this junction is called E-Plane Tee junction. This is also called as **Voltage** or **Series junction**. The ports 1 and 2 are 180° out of phase with each other. The cross-sectional details of E-plane tee can be understood by the following figure. An E-Plane Tee junction is designed by assigning a simple waveguide to the broader dimension of a rectangular waveguide, which previously has two ports. The arms of rectangular waveguides create two ports called collinear ports i.e. Port1 and Port2, while the new one, Port3 is called as Side arm or E-arm. T his E- plane Tee is also called as Series Tee.

As the axis of the side arm is similar to the electric field, this junction is called E-Plane Tee junction. This is also called as Voltage or Series junction. The ports 1 and 2 are 180° out of phase with each other. The cross-sectional details of E-plane tee can be assumed by the resulting figure.



The resulting figure displays the connection made by the sidearm to the bi-directional waveguide to form the parallel port.



Properties of E-Plane Tee

The properties of E-Plane Tee can be defined by its $[S]_{3x3}[S]_{3x3}$ matrix.

1.It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{........ Equation 1}$$

2.Scattering coefficients S_{13} and S_{23} are out of phase by 180° with an input at port 3

$$S_{23} = -S_{13} \quad \text{........ Equation 2}$$

3.The port is perfectly matched to the junction.

-

$$S_{33} = 0 \quad \text{........ Equation 3}$$

4. From the symmetric property,

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} \;\; S_{23} = S_{32} \;\; S_{13} = S_{31}$$ **........ Equation 4**

Considering equations 3 & 4, the [S] matrix can be written as,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$ **........ Equation 5**

We can say that we have four unknowns, considering the symmetry property.

5. From the Unitary property

$$[S][S]* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying we get,

(Noting R as row and C as column)

$$R_1C_1 : S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{11}|^2 + |S_{11}|^2 = 1$$ **........ Equation 6**

$$R_2C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$ **......... Equation 7**

$$R_3C_3 : |S_{13}|^2 + |S_{13}|^2 = 1$$ **......... Equation 8**

$$R_3C_1 : S_{13}S_{11}^* - S_{13}S_{12}^* = 1$$ **......... Equation 9**

Equating the equations 6 & 7, we get

$$S_{11} = S_{22}$$ **......... Equation 10**

From Equation 8,

$$2|S_{13}|^2 \quad or \quad S_{13} = \frac{1}{\sqrt{2}}$$ **......... Equation 11**

From Equation 9,

$$S_{13}\left(S_{11}^{*}-S_{12}^{*}\right)$$

Or $S_{11}=S_{12}=S_{22}$ **......... Equation 12**

Using the equations 10, 11, and 12 in the equation 6,

we get,

$$|S_{11}|^{2}+|S_{11}|^{2}+\tfrac{1}{2}=1$$

$$2|S_{11}|^{2}=\tfrac{1}{2}$$

Or $S_{11}=\frac{1}{2}$ **......... Equation 13**

Substituting the values from the above equations in [S][S] matrix,

We get,

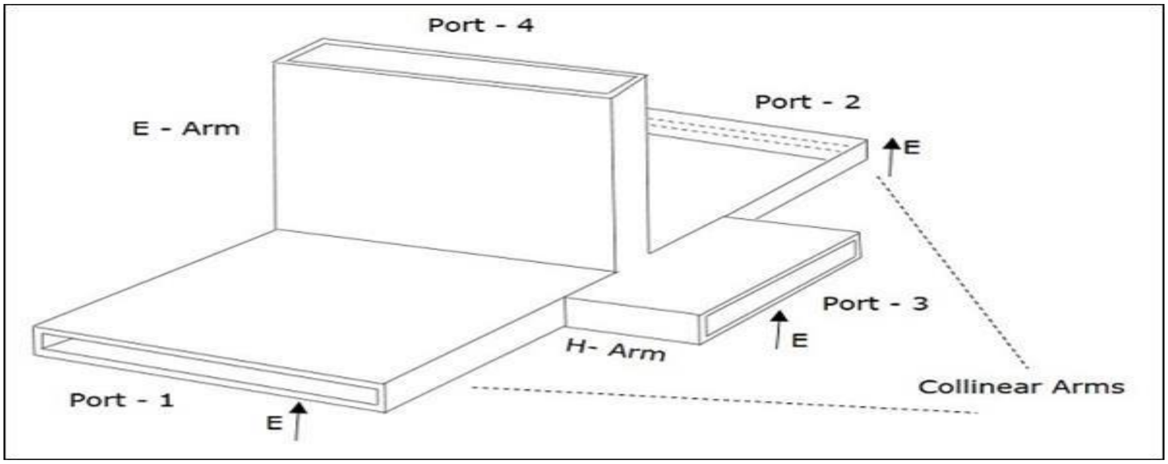
$$[S]=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0\end{bmatrix}$$

We know that [b]= [S][a]

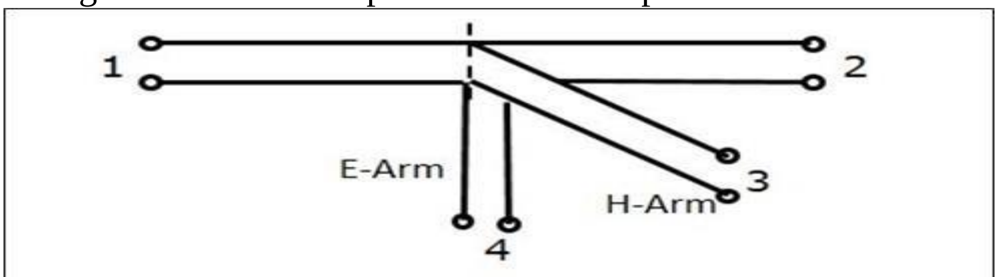
$$\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}$$

This is the scattering matrix for E-Plane Tee, which explains its scattering properties.

E-H-Plane



The resulting figure shows the assembly made by the side arms to the bi-directional waveguide to form both parallel and serial ports.



Magic Tee junction

Characteristics of E-H Plane Tee

- If a signal of equal phase and magnitude is sent to port 1 and port 2, then the output at port 4 is zero and the output at port 3 will be the additive of both the ports 1 and 2.
- If a signal is sent to port 4, (E-arm) then the power is divided between port 1 and 2 equally but in opposite phase, while there would be no output at port 3. Hence, $S_{34} = 0$.
- If a signal is fed at port 3, then the power is divided between port 1 and 2 equally, while there would be no output at port 4. Hence, $S_{43} = 0$.
- If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port, as the E-arm produces a phase delay and the H-arm produces a phase advance. So, $S_{12} = S_{21} = 0$.

Properties of E-H Plane Tee

The properties of E-H Plane Tee can be defined by its $[S]_{4\times4}$matrix.

1. It is a 4×4 matrix as there are 4 possible inputs and 4 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$ **........ Equation 1**

2. As it has H-Plane Tee section As it has H-Plane Tee section

$$S_{23} = S_{13}$$ **........ Equation 2**

3. As it has E-Plane Tee section

$$S_{24} = -S_{14}$$ **........ Equation 3**

4. The E-Arm port and H-Arm port are so isolated that the other won't deliver an output, if an input is applied at one of them. Hence, this can be noted as

$$S_{34} = S_{43} = 0$$ **........ Equation 4**

5. From the symmetry property, we have

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{14} = S_{41}$$

$$S_{23} = S_{32}, S_{24} = S_{42}, S_{34} = S_{43}$$ **........ Equation 5**

6. If the ports 3 and 4 are perfectly matched to the junction, then

$$S_{33} = S_{44} = 0$$ **........ Equation 6**

Substituting all the above equations in equation 1, to obtain the [S][S] matrix,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$ **........ Equation 7**

7. From Unitary property, $[S][S]* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 : |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 + |S_{14}|^2 = 1$ **......... Equation 8**

$R_2C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 + |S_{14}|^2 = 1$ **......... Equation 9**

$R_3C_3 : |S_{13}|^2 + |S_{13}|^2 = 1$ **......... Equation 10**

$R_4C_4 : |S_{14}|^2 + |S_{14}|^2 = 1$ **......... Equation 11**

From the equations 10 and 11, we get

$S_{13} = \frac{1}{\sqrt{2}}$ **........ Equation 12**

$S_{14} = \frac{1}{\sqrt{2}}$ **........ Equation 13**

Comparing the equations 8 and 9, we have

$S_{11} = S_{22}$ **......... Equation 14**

Using these values from the equations 12 and 13, we get

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$S_{11} = S_{22} = 0$ **......... Equation 15**

From equation 9, we get

$S_{22} = 0$ ....**Equation 16**

Now we understand that ports 1 and 2 are perfectly matched to the junction. As this is a 4 port junction, whenever two ports are perfectly matched, the other two ports are also perfectly matched to the junction.

The junction where all the four ports are perfectly matched is called as Magic Tee Junction.

By substituting the equations from 12 to 16, in the [S][S] matrix of equation 7, we obtain the scattering matrix of Magic Tee as

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

We already know that, [b] = [S][a]

Rewriting the above, we get

$$\begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{vmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{vmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{vmatrix}$$

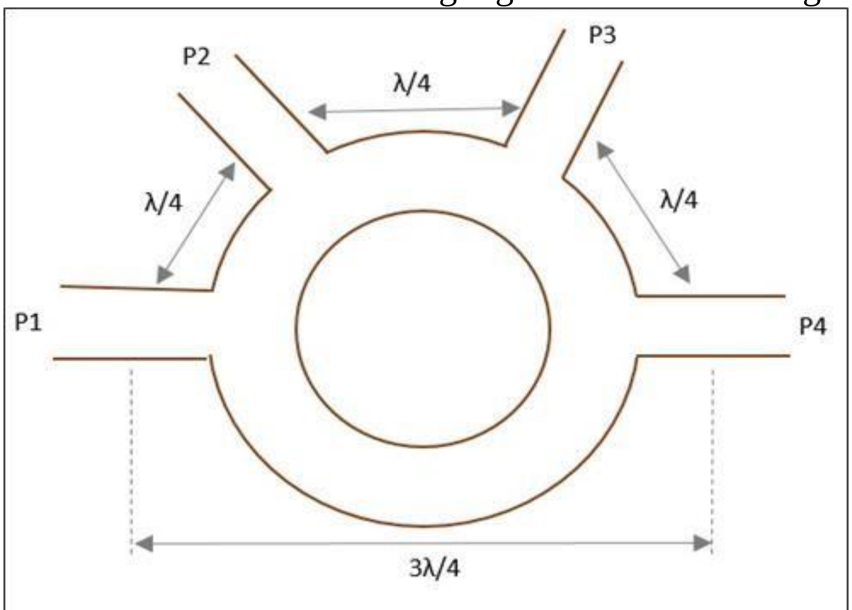
Applications of E-H Plane Tee

Some of the greatest mutual applications of E-H Plane Tee are as follows :

- E-H Plane junction is used to amount the impedance − A null detector is linked to E-Arm port while the Microwave source is linked to H-Arm port. The collinear ports composed with these ports make a bridge and the impedance measurement is done by balancing the bridge.
- E-H Plane Tee is used as a duplexer − A duplexer is a circuit which mechanisms as both the transmitter and the receiver, by means of a single antenna for both drives. Port 1 and 2 are used as receiver and transmitter where they are inaccessible and hence will not interfere. Antenna is connected to E-Arm port. A matched load is connected to H- Arm port, which provides no reflections. Currently, there exists transmission or reception without any problem.
- E-H Plane Tee is used as a mixer − E-Arm port is connected with antenna and the H-Arm port is connected with local oscillator. Port 2 has a matched load which has no

reflections and port 1 has the mixer circuit, which gets half of the signal power and half of the oscillator power to produce IF frequency.

- In addition to the above applications, an E-H Plane Tee junction is also used as Microwave bridge, Microwave discriminator, etc.
- If we need to association two signals with no phase modification and to avoid the signals with a path difference then we need microwave device. A usual three-port Tee junction is taken and a fourth port is added to it, to make it a ratrace junction. All of these ports are linked in angular ring forms at equal intervals using series or parallel junctions.
- The mean circumference of total race is 1.5λ and each of the four ports is detached by a distance of λ/4. The resulting figure shows the image of a Rat-race junction.



Rat-race Coupler

Let us study a few cases to appreciate the operation of a Rat-race junction.

Case 1

If the input power is applied at port 1, it gets similarly split into two ports, but in clockwise direction for port 2 and anti-clockwise direction for port 4. Port 3 has unconditionally no output.The reason being, at ports 2 and 4, the powers combine in phase, whereas at port 3, cancellation occurs due to λ/2 path difference.

Case 2

If the input power is applied at port 3, the power gets similarly separated between port 2 and port 4. But there will be no output at port 1.

Case 3

If two unequal signals are applied at port 1 itself, then the output will be relative to the sum of the two input signals, which is separated between port 2 and 4. Now at port 3, the differential output appears.

The Scattering Matrix for Rat-race junction is represented as

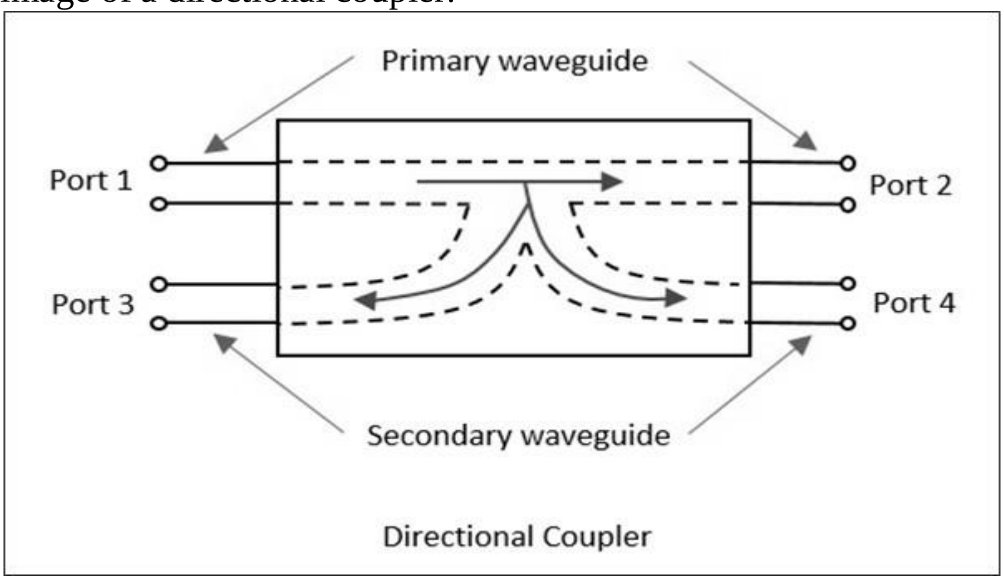
$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Applications:

Rat-race junction is used for uniting two signals and separating a signal into two halves.

Directional coupler

- A Directional coupler is a device that trials a minor amount of Microwave power for measurement tenacities. The power measurements comprise incident power, reflected power, VSWR values, etc.
- Directional Coupler is a 4-port waveguide junction comprising of a primary main waveguide and a secondary supporting waveguide. The resulting figure shows the image of a directional coupler.

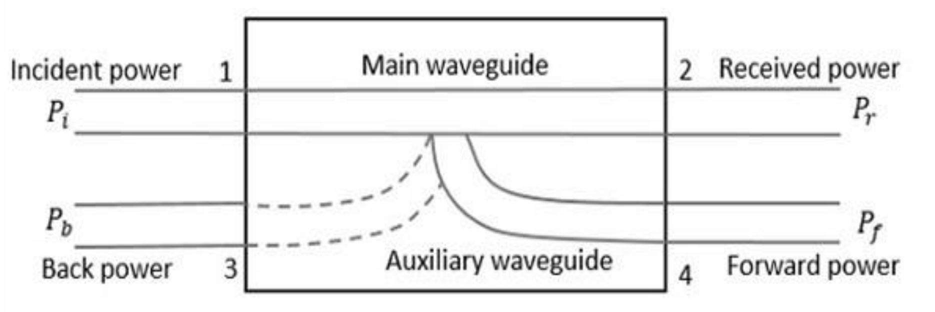


- Directional coupler is used to couple the Microwave power which may be unidirectional or bi-directional.

Properties of Directional Couplers:

The properties of an ideal directional coupler are as follows.

- All the finishes are matched to the ports.
- When the power travels from Port 1 to Port 2, some portion of it gets coupled to Port 4 but not to Port 3.
- As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.
- If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.
- If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2.
- Port 1 and 3 are decoupled as are Port 2 and Port 4.
- Preferably, the output of Port 3 should be zero. Though, almost, a small amount of power called back power is practical at Port 3. The resulting figure specifies the power flow in a directional coupler.



Directional Coupler indicating powers

Where

- P_i = Incident power at Port 1
- P_r = Received power at Port 2
- P_f= Forward coupled power at Port 4
- P_b = Back power at Port 3

Resulting are the parameters used to define the performance of a directional coupler.

Coupling Factor (C)

The Coupling factor of a directional coupler is the ratio of incident power to the forward power, measured in dB.

$$C = 10\, log_{10} \frac{P_i}{P_f} dB$$

Directivity (D)

The Directivity of a directional coupler is the ratio of forward power to the back power, measured in dB.

$$D = 10\, log_{10} \frac{P_f}{P_b} dB$$

Isolation

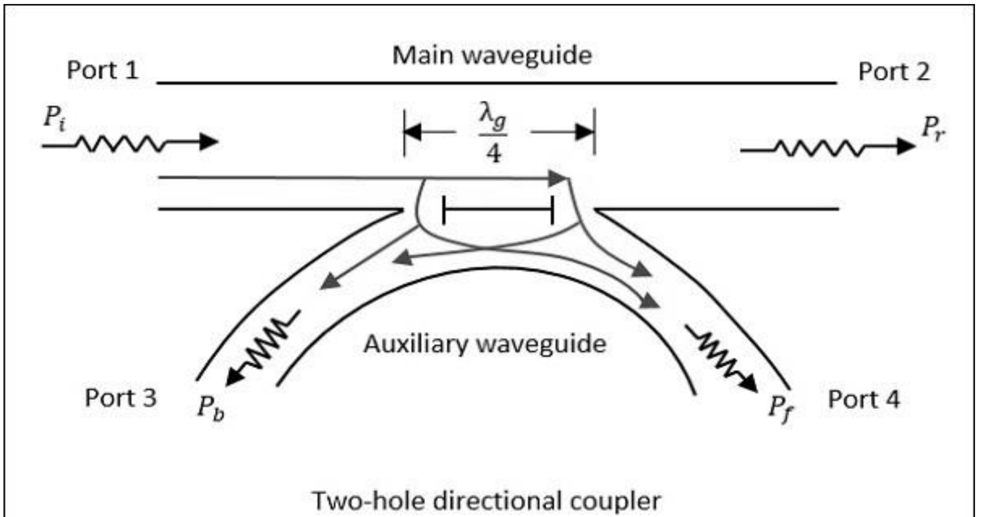
It defines the directive properties of a directional coupler. It is the ratio of incident power to the back power, measured in dB.

$$I = 10\, log_{10} \frac{P_i}{P_b} dB$$

Isolation in dB = Coupling factor + Directivity

Two-Hole Directional Coupler

This is a directional coupler with same main and auxiliary waveguides, but with two small holes that are common between them. These holes are λg/4 distance apart where λg is the guide wavelength. The following figure shows the image of a two-hole directional coupler.



Two-hole directional coupler

A two-hole directional coupler is planned to see the ideal condition of directional coupler, which is to evade back power. Some of the power while travelling between Port 1 and Port 2, escapes through the holes 1 and 2.

The greatness of the power depends upon the dimensions of the holes. This leakage power at both the holes are in phase at hole 2, adding up the power causal to the forward power Pf. Though, it is out of phase at hole 1, stopping each other and stopping the back power to occur. Therefore, the directivity of a directional coupler improves. The general S matrix of a directional coupler is,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} ------(1)$$

1. Since all ports in a directional coupler are matched.

$S_{11} = S_{22} = S_{33} = S_{44} = 0$ -------------------(2)

2. Since there is no coupling between ports 1 & 3 and ports 2 & 4

$S_{13} = S_{31} = S_{24} = S_{42} = 0$ --------------(3)

Apply equation (2) & (3) in (1)

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

3. By unitary property, [S][S]* = I

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow |S_{12}|^2 + |S_{14}|^2 = 1$ -------------(4)
$R_2C_2 \Rightarrow |S_{12}|^2 + |S_{23}|^2 = 1$ -------------(5)
$R_3C_3 \Rightarrow |S_{23}|^2 + |S_{34}|^2 = 1$ -------------(6)
$R_1C_3 \Rightarrow S_{12} S_{23}^* + S_{14} S_{34}^* = 0$------------ (7)

Comparing equations (4) and (5)

$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$
$S_{14} = S_{23}$---------------- (8)

Comparing equations (5) and (6)

$|S_{12}|^2 + |S_{23}|^2 = |S_{34}|^2 + |S_{23}|^2$
$S_{12} = S_{34}$---------------- (9)

Let, S_{12} be real and positive,
i.e, $S_{12} = S_{34} = p$ ----------------(10)

applying equation (10) in (7)

Therefore, $p\, S_{23}^* + S_{14}\, p = 0$
$p\, [S_{23}^* + S_{14}] = 0$
$p\, [S_{23}^* + S_{23}] = 0$
$S_{23}^* + S_{23} = 0$

To satisfy the above condition, S_{23} should be a complex value.

Let $S_{23} = jq$

Therefore, the S matrix of directional coupler is,

$$S = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

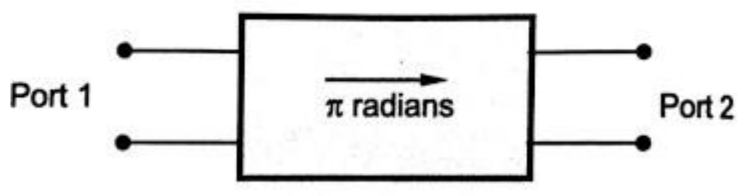
Ferrite components:

Introduction to ferrites:

A ferrite is a nonmetallic material (though often an iron oxide compound) which is an insulator, but with magnetic properties similar to those of ferrous metals. Among the more common ferrites are manganese ferrite ($MnFe_2O_3$), zinc ferrite ($ZnFe_2O_3$) and associated ferromagnetic oxides such as yttrium-iron-garnet [$Y_3Fe_2(FeO_4)_3$], or YIG for short. (Gamets are vitreous mineral substances of various colors and composition, several of them being quite valuable as gems.) Since all these materials are insulators, electromagnetic waves can propagate in them. Because the ferrites have strong magnetic properties, external magnetic fields can be applied to them with several interesting results, including the Faraday rotation mentioned in connection with wave propagation.

When electromagnetic waves travel through a ferrite, they produce an RF magnetic fieldin the material, at right angles to the direction of propagation if the mode of propagation is correctly chosen. If an axial magnetic field from a permanent magnet is applied as well, a complex interaction takes place in the ferrite. The situation may be somewhat simplified if weak and strong interactions are considered separately.

Gyrator:



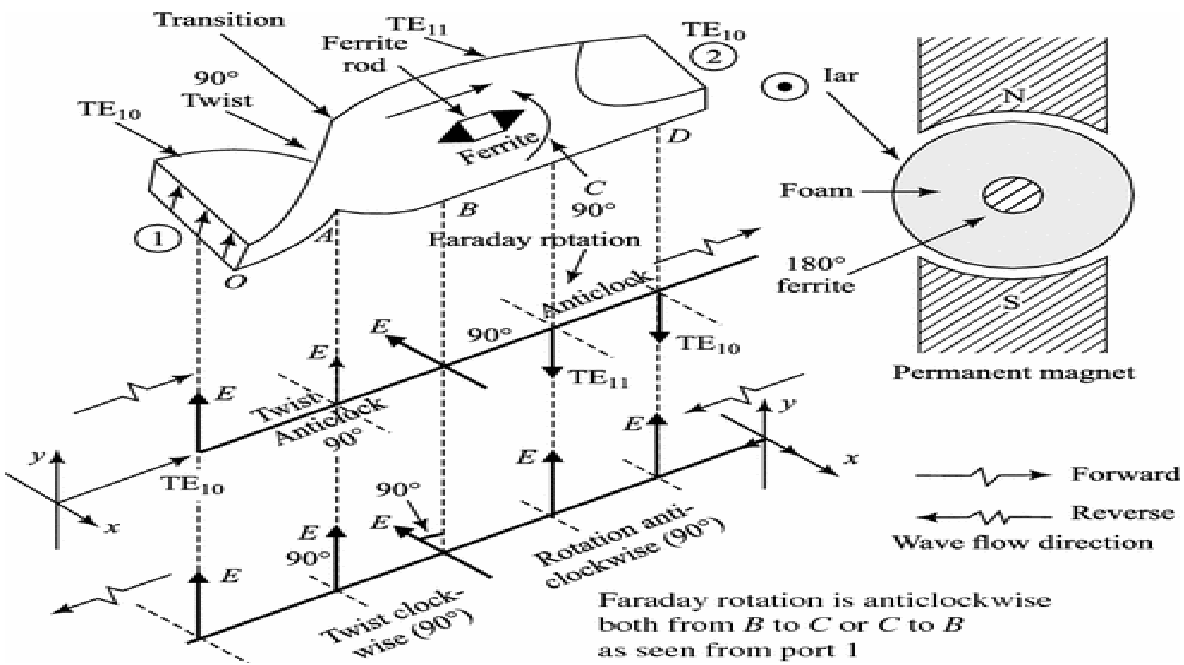
- Gyrator is two port device that has the relative phase difference of 180º for transmission from port 1 to port 2 and "no phase shift" for transmission from port 2 to port 1.

Construction:

- It consists of a piece of circular waveguide carrying the dominant TE_{11} mode with transitions to a standard rectangular waveguide with dominant mode TE_{10} at both ends.
- A thin circular ferrite rod tapered at both ends is located inside the circular waveguide supported by a polyfoam.
- The waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite.
- To the input end a 90º twisted rectangular waveguide is connected.
- The ferrite rod is tapered at both ends to reduce the attenuation and also for smooth rotation of polarized wave.

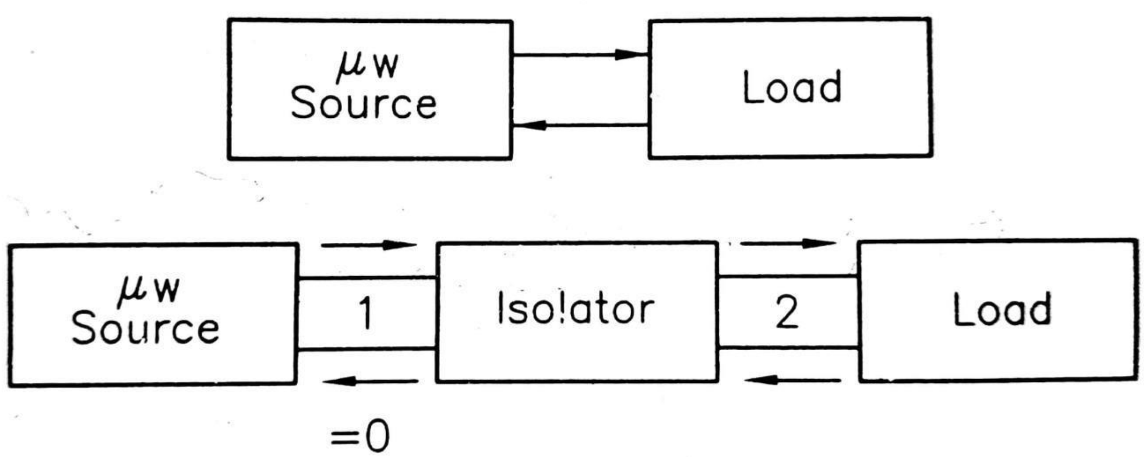
Operation

- When a wave enters port 1 its plane of polarization rotates by 90º because of the twist in the waveguide.
- It again undergoes faraday rotation through 90º because of the ferrite rod and the wave which comes out of port 2 will have a phase shift of 180º compared to the wave entering at port 1.
- When the same wave enters at port 2, it undergoes faraday rotation through 90º in the same direction.
- Because of the twist this wave gets rotated back by 90º comes out of port 1 with 0º phase shift.



Isolators:

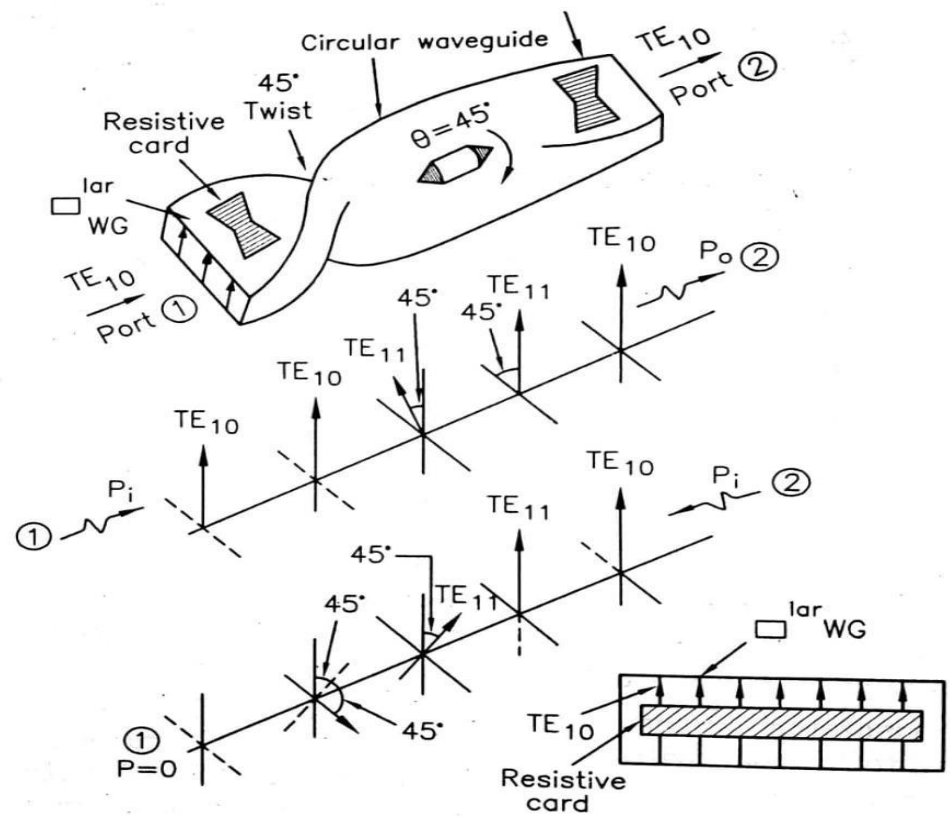
- An isolator is a 2-port device which provides a very small amount of attenuation for transmission from port 1 to port 2 but provides maximum attenuation for transmission from port 2 to port 1.
- This requirement is very much desirable when we want to match a source with a variable load.
- In most microwave generators, the output amplitude and frequency tend to fluctuate very significantly with changes in load impedance.
- Due to mismatch of generator output to the load resulting in reflected wave from load.
- These reflection will cause amplitude and frequency instabilities of the microwave generator.
- When the isolator is inserted between generator and load, the generator is coupled to the load with zero attenuation and if any reflection from the load is completely absorbed by the isolator without affecting the generator output.



Construction

- Isolator makes use of 45º twisted rectangular waveguide and 45º faraday rotation ferrite rod.
- A resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb any wave whose plane of polarization is parallel to the plane of resistive card.

- The resistive card does not absorb any wave whose plane of polarization is perpendicular to the plane of its own.



Operation

- A TE_{10} wave passing from port 1 through the resistive card and is not attenuated.
- After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction and then by another 45° in clockwise direction because of the ferrite rod and hence coming out of port 2 with the same polarization as at port 1 without any attenuation.
- But a TE_{10} wave fed from port 2 gets a pass from the resistive card placed near at port 2 because plane of polarization of the wave is perpendicular to the plane of the resistive card.

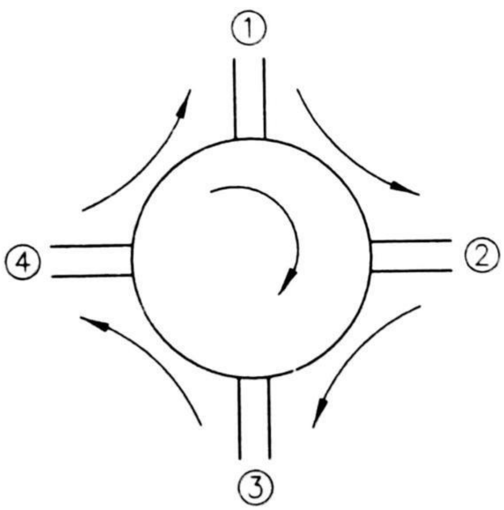
- Then the wave gets rotated by 45º in clockwise direction due to ferrite rod and rotated by another 45º due to the twist in the waveguide.
- Now the plane of polarization of the wave is parallel with the plane of resistive card and hence the wave will be completely absorbed by the resistive card and the output at port 1 will be zero.
- This power is dissipated in the card as a heat.
- In practice 20 to 30 dB isolation is obtained for transmission from port 2 to port 1.

Circulator:

- A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal i.e., port 1 is connected to port 2 only and not to port 3 and port 4.
- Although there is no restriction on the number of ports, four ports are most commonly used.

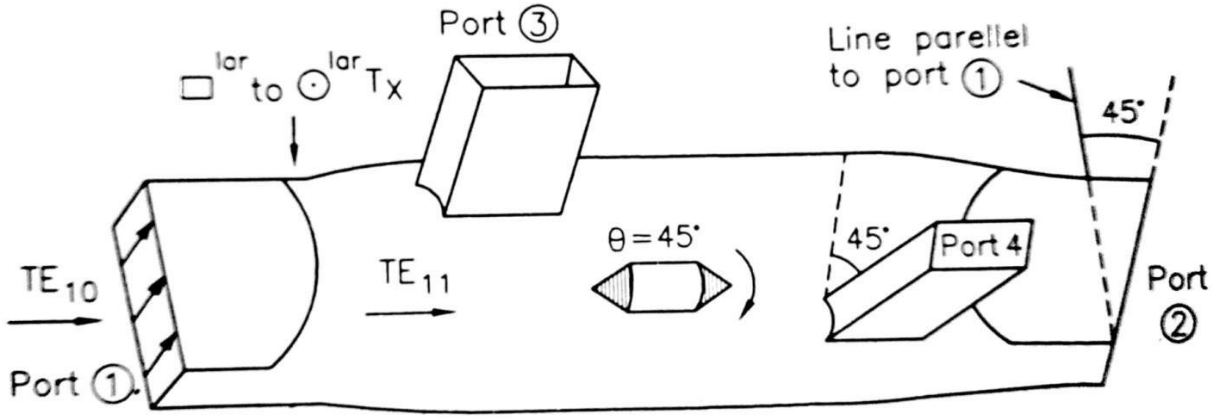
Operation

- The power entering port 1 is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition.
- This power passes port 3 unaffected since the electric filed is not significantly cut and is rotated through 45ºdue to the ferrite, passes port 4 unaffected for the same reason and finally emerges out of port 2.
- Power from port 2 will have plane of polarization already titled by 45º with respect to port 1.



- **This power passes port 4 unaffected and gets rotated by 45º due to ferrite rod in the clockwise direction. And now totally plane of polarization is tilted through 90º finds port 3 suitably aligned and emerges out of it.**

- **Similarly port 3 in coupled only to port 4 and port 4 to port 1.**


Port ③
□ lar to ⊙ lar Tx
Line parellel
to port ①
45°
45°
Port 4
$\theta = 45°$
TE_{10}
TE_{11}
Port
②
Port ①

UNIT III
Linear beam Tubes

Contents:

- Limitations and losses of conventional tubes at microwave frequencies
- Classification of Microwave tubes
- **type tubes** – 2 cavity klystron
  - structure
  - velocity modulation process and Applegate diagram
  - bunching process
  - small signal theory Expressions for o/p power
  - efficiency,
- Reflex Klystrons-
  - Structure
  - Velocity Modulation
  - Applegate diagram
  - Power output
  - Efficiency.

MICROWAVE TUBES

Limitations and losses of conventional Tubes at Microwave Frequencies

Conventional vacuum triodes, tetrodes and pentodes are less useful signal sources at frequencies above 1 G Hz because of

- lead inductance
- Inter-electrode capacitance effects,
- Transit angle effects
- Gain bandwidth product limitations.
- Power losses

Lead inductance and inter-electrode capacitance effects

At frequencies above 1 GHz conventional vacuum tubes are impaired by parasitic circuit reactance because the circuit capacitances between tube electrodes and the circuit inductance of the lead wire are too large for a microwave resonant circuit. Further as the frequency increases the real part of the input admittance may be large enough to cause a serious over load of the input circuit and thereby reduce the operating efficiency of the tube.

Transit angle effects

Another limitation in the application of conventional tubes at microwave frequencies is the electron transit angle between electrodes. The electron transit angle is defines as

$$\Theta_g = \omega\tau = \frac{\omega d}{v_o}$$

Where $\tau = \frac{d}{v_o}$ is the transit time across the gap

d = separation between cathode and grid
v_o = Velocity of the electron $0.593 \times 10^5 \sqrt{V_o}$
V_o = DC voltage

When frequencies are below microwave range, the transit angle is negligible. At microwave frequencies, however the transit time is large compared to the period of the microwave signal, and the potential between the cathode and the grid mayalternate from 10 to 100 times during the electron transit. The grid potential duringthe negative half cycle thus removes energy that was given to the electron during the positive half cycle. Consequently, the electrons may oscillate back and forth in the cathode-grid space or return to the cathode. The overall result of transit angle effects is to reduce the operating efficiency of the vacuum tube. The degenerate effect becomes more serious when frequencies are well above 1 GHz.

Gain bandwidth product limitations

The gain-bandwidth product is independent of frequency. For a given tube, a higher gain can be achieved only at the expense of a narrower bandwidth. This restriction is applicable to a resonant circuit only. In microwave devices either reentrant cavities or slow-wave structures are used to obtain a possible overall high gain over a bandwidth.

Power losses

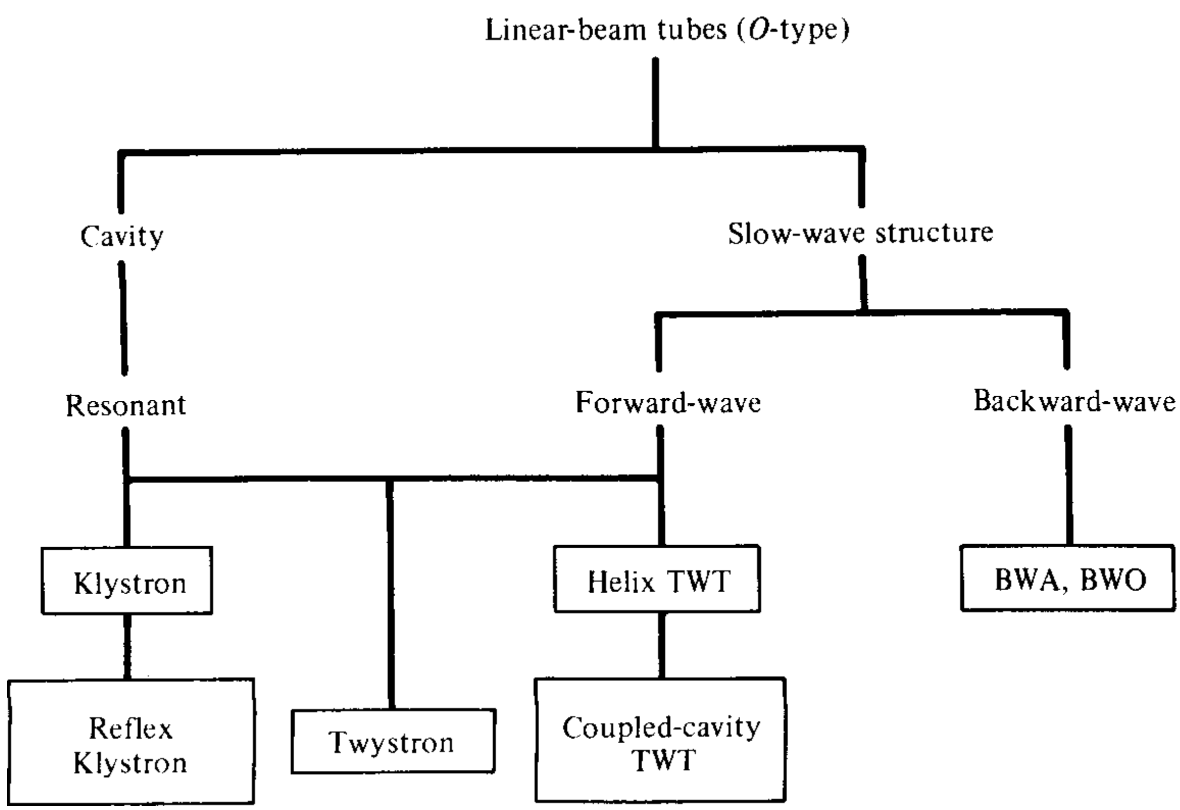
The use of conventional tubes at higher frequencies also increases in power losses resulting from skin effect, I^2R losses resulting from capacitance charging currents, losses due to radiation from the circuit and dielectric losses.

Classification of Microwave tubes.

Microwave Tubes

O-Type Tubes (Linear beam tubes)

M-Type Tubes (Cross field tubes)



Energy Transfer Mechanism

In most of the microwave tubes, the signal is placed in a cavity gap and electrons are forced to cross the gap at time when they face maximum opposition. Crosssing the gap under opposition lead to transfer of energy to the cavity gap signal. When thegap voltage is sinusoidal time-varying and the charge corssing is continuous and uniform, which is usually the case, no net transfer of energy takes place between cavity and the charge crossing the gap. It is because the energy transfer is equaland opposite in direction during a half cycle when compalred to previous half cycle resulting in no net transfer of energy in a cycle. To have net energy transfer, preferabley maximum, from electron beam to gap signal voltage the disributed charge is compressed into a thin sheet or bunch, so that it requires less time to cross the gap and it is arranged such hat the bunch corssing is at peak gap voltage so that the bunch faces maximum opposition and retardation from the signal voltage.

When the gap voltage is simusoidal and bunch corssing is at a uniform and constant rate, for maximum unidirectional flow of energy, there is only one instant, either at positive peak or negative peak, for the bunch to cross the gap. The bunch crossing hence mus be once per cycle of the gap voltage. In cse of bunch corssing at a uniform rate of ***f***, transfer of maximum energy can take place only with a a component of grid gap field whose frequency is also ***f***. Other components of the grid gap voltge like 2***f***, 4***f***, 8***f***, ets., do not involve in the energy transfer, whereas the components 3***f***, 5***f***, 6***f***, etc., and ***f***/2, ***f***/3. ***f***/4, etc., the transfered amount of energy is negligible.

Two Cavity Klystron

The two-cavity klystron is a widely used **microwave amplifier** operated by the principles of velocity and current modulatin. It consists of an elctron gun, focussing and accelerating grids, two identical cavities separated by a distance and at the far end a gounded collector plaate. The elctron gun emits electrons from the surace of itscathode and then they are focussed into a bearm. Using a dc accelerating positive voltage the beam is accelerated to high velocities.

Characteristics of two cavity klystron

Efficiency:	about 40%
Power output in CW mode:	upto 500 kW at 10 GHz
Power output in Pusled mode:	upto 30 MW at 10 GHz
Power gain:	about 30 dB

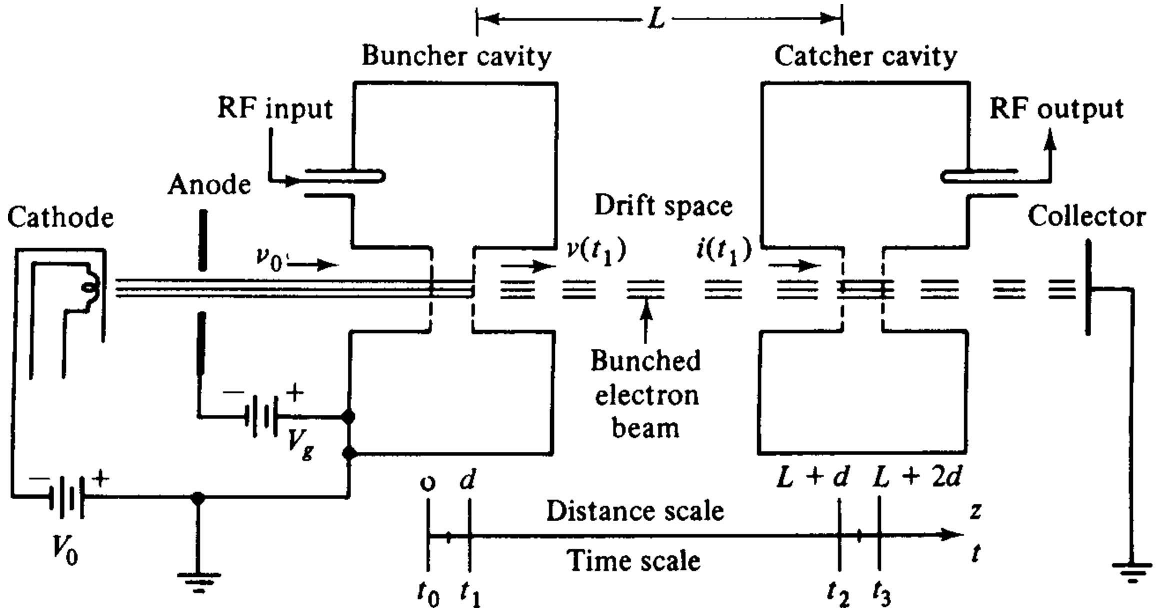


Fig 5.1 Scematic diagram of two cavity klystron

Components of two cavity klystron

1. **Cathode:** Source of electrons
2. **Anode:** for formation of electron beam
3. **Bucher Cavity:** A reentrat type resonator cavity which is kept at a +ve voltage of V_o w.r.t. cathode to effect acceleration of electrons. RF input voltage of $\mathbf{V_1 \sin \omega t}$ is applied to buncher cavity.
4. **Catcher cavity:** Similar to buncher cavity. The amplified output signal $\mathbf{V_2 \sin \omega t}$ is obtained from this cavity.
5. **Collector:** The elctrons after transfer of enerty to catcher cavity are collected by the collector.

Let us define varous parameters used in the description and operation of two cavity klystron.

$\mathbf{V_o}$ = DC voltage between cathode and buncher cavity.

$\mathbf{V_1}$ = Amplitude of input RF signal, $\mathbf{V_{1} << V_o}$

$\boldsymbol{\omega = 2\pi f}$ = Input signal angular frequency. It is also equal to resonant frequency of both the cavities.

v_o = Uniform velocity of electrons between cathode and buncher cavity.

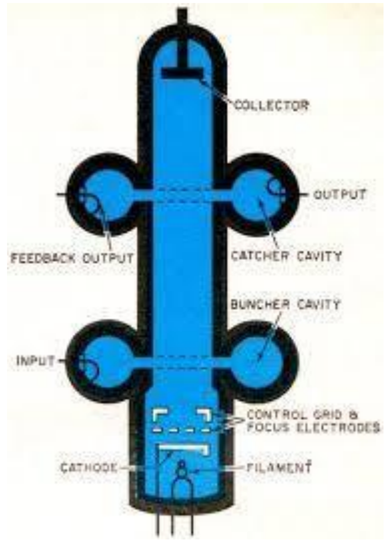
t_o = Time at which electrons enter the buncher cavity

t_1 = Time at which electrons leave the buncher cavity

τ = Transit time of electrons in the buncher cavity = $t_1 - t_o$

θ_g = Angle /Phase vaiation of input signal during the tansit time = ωŤ

β = Beam Coupling Coefficient of the buncher / Catcher Cavity



When the electrons enter the buncher cavity with uniform velocity „v_0" interact with the field due to input RF signal V_1sin ωt. The time varying field in the cavity cause the electrons to accelerate or decelerate and there by electrons undergo velocity modulation.

Let **v**(t_1) = Velocity of electrons at t= t_1 at the output of buncher cavity

This is a time varying quantity

Refer fig-5.3 for velocity modulation of electrons

Let d = cavity with of buncher / catcher cavity

L = spacing between buncher and catcher cavities

* „L" is the design parameter for optimum performance of the klystron amplifier

t_2 = Time at which electrons enter the catcher cavity

t_3= Time at which electrons leave the catcher cavity

$$\tau \approx \frac{d}{v_o} = t_1 - t_0 \qquad (5.2)$$

Because $V_1 << V_0$

<u>Evaluation of *v*(t_1)</u>

***v*(t_1)** is the instantaneous velocity of electrons which is a time varying quantity and primarily depends upon the average voltage in the gap during the time „τ" i.e. during the time period (transit) the electrons are influenced by the field.

V_{avg} = Average voltage in the gap during time „τ"

$$V_{avg} = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin\omega t \, dt$$

$$= \frac{-V_1}{\omega\tau}\left[\cos\omega t\right]_{t_0}^{t_1}$$

$$V_{avg} = \frac{-V_1}{\omega\tau}\left[\cos\omega t_1 - \cos\omega t_0\right] \quad = \quad V_{avg} = \frac{V_1}{\omega\tau}\left[\cos\omega t_o - \cos\omega t_1\right]$$

Where $\tau = t_1 - t_0$

$t_1 = t_0 + \tau = t_0 + \dfrac{d}{v_o}$

$$V_{avg} = \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right]$$

Where $\theta_g = \omega\tau = \dfrac{\omega d}{v_0}$ (5.3)

$$V_{avg} = \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos(\omega t_0 + \theta_g)\right]$$

Let $A = \omega t_0 + \dfrac{\theta_g}{2}$

and $B = \dfrac{\theta_g}{2}$

Since cos (A-B) – cos (A+B) = 2 sin A sin B

$$V_{avg} = V_1\left\{\frac{\sin(\omega d/2v_0)}{\omega d/2v_0}\right\}\sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right)$$

$$= V_1 \frac{\sin\theta_g/2}{\theta_g/2}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

β_i = beam coupling coefficient of input (buncher) cavity by definition

$$\beta_i = \frac{\sin\theta_g/2}{\theta_g/2} \quad (5.4)$$

$$V_{avg} = V_1\beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

As we have seen earlier $v_0 = \sqrt{\dfrac{2eV_0}{m}}$

|||ly $\boldsymbol{v(t_1)} = \; = \sqrt{\dfrac{2e}{m}}.\sqrt{V_1\beta_i \sin\left(\omega t_0 + \dfrac{\theta_g}{2}\right) + V_0}$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2} \right) \right]}$$

Since $V_1 << V_0$, $\frac{\beta_i V_1}{V_0} << 1$

Using binomial expansion $\sqrt{1+x} = 1 + \frac{x}{2}$ *for* $x << 1$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

Since $\tau = t_1 - t_0$

$\theta_g = \omega\tau = \omega t_1 - \omega t_0$

$\omega t_0 = \omega t_1 + \omega t_0$

$\omega t_0 + \theta_g/2 = \omega t_1 - \theta_g + \theta_g/2 = \omega t_1 - \theta_g/2$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(wt_1 - \frac{\theta_g}{2}\right)\right] \tag{5.5}$$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(wt_0 + \frac{\theta_g}{2}\right)\right] \tag{5.6}$$

<u>Numerical :</u> *The parameters of a 2 cavity klystron are*

v_0 = 1000 V, *I_0 = 25 mA(beam current)*

d = 1mm, *f = 3 GHz*

Find out β_i beam coupling coefficient

Solution $v_0 = \frac{0.593\times10^6}{\sqrt{V_0}} = 0.593\times10^6 \times \sqrt{1000}$

= 1.88 x 10⁷ m/s

$$\theta_g = \frac{\omega d}{v_0} = \frac{2\pi\times3\times10^9\times10^{-3}}{1.88\times10^7} = 1.002\ rad$$

$$\beta_i = \frac{\sin\theta_g/2}{\theta_g/2} = \frac{\sin 0.5}{0.5} = 0.958$$

Bunching Process of Electrons

All the electrons in the beam will drift with a uniform velocity of " v_0 "at t $=t_0$ i.e. at time of entry into the buncher cavity. For $t_2 > t > t_0$ i.e. in the cavity gap the velocity of electrons vary with time depending upon the instantaneous field $V_1 \sin wt$

KLYSTRON

SIMPLIFIED SCHEMATIC

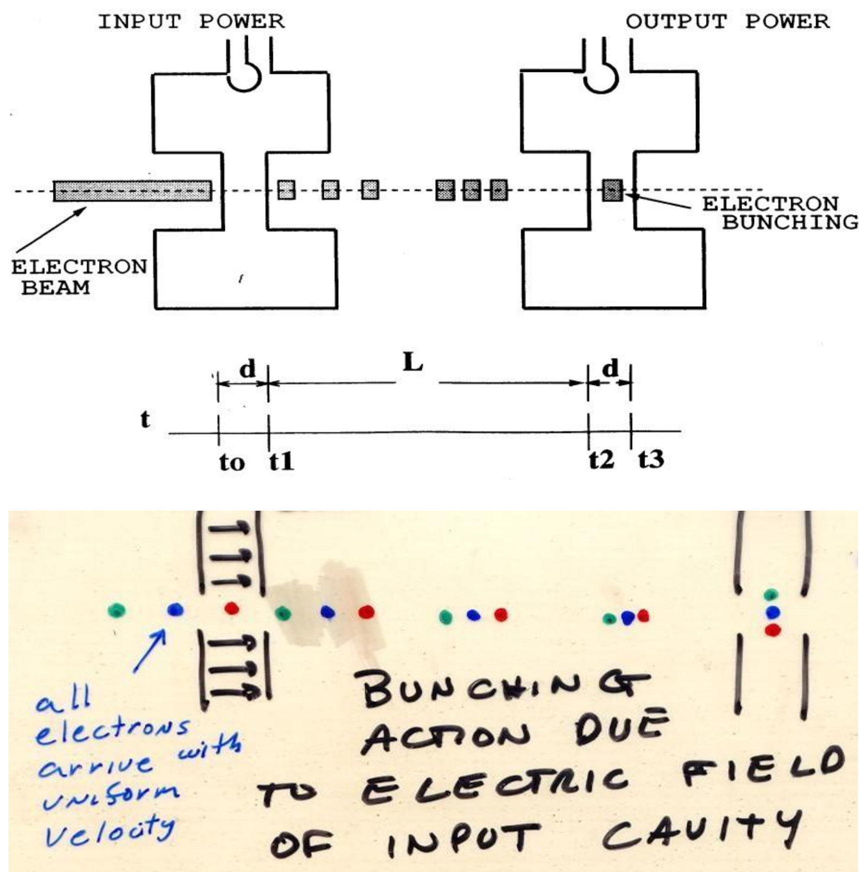


Fig 5.2: Bunching process in 2-cavity klystron

Consider three arbitrary electrons a, b and c passing thro the gap when the field is –ve max, zero and +ve max respectively at time instances t_a, t_b and t_c.

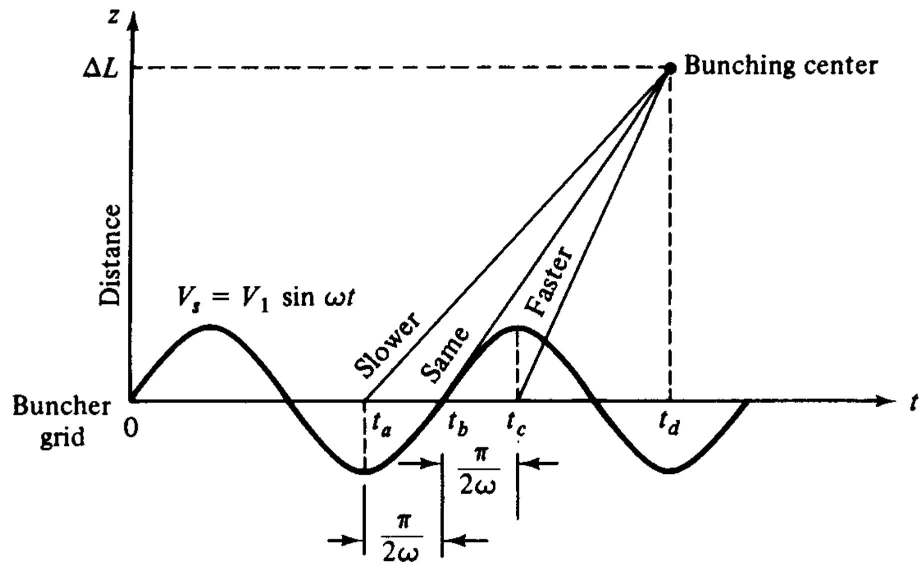


Fig 5.3 : Appligate diagram

Velocity of electron „b" = v_b = v_0 (field zero)

Velocity of electron „a" = $v_a < v_0$ = $v_{\min}$ (-ve field)

Velocity of electron „c" = $v_c > v_0$ = v_0 = $v_{\max}$ (+ve field)

Let us consider that these three electrons draft with different velocities and meet (bunch) together at t = t_d at a length ΔL from buncher cavity.

$$\Delta L = v_{\min} (t_d - t_a) \qquad (5.7)$$

$$\Delta L = v_0 (t_d - t_b) \qquad (5.8)$$

$$\Delta L = v_{\max} (t_d - t_c) \qquad (5.9)$$

$t_c - t_b = t_b - t_a = \pi / 2w$ (1/4 of time period) (5.10)

$\Delta L = v_{\min} (t_d - t_a) = v_{\min} (t_d - t_b + \pi/2\omega)$ (5.7A)

$\Delta L = v_{\max} (t_d - t_c) = v_{\max} (t_d - t_b - \pi/2\omega)$ (5.8A)

We have $v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$

From above equation

$$v_{\max} = v(t_1) = v_0\left(1 + \frac{\beta_i V_1}{2V^0}\right) \qquad (5.11)$$

$$v_{min} = v(t_1) = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right) \tag{5.12}$$

Substituting equation 12, 11 in equation 7A and 8A

$$\Delta L = v_0(t_d - t_b) + \left[v_0\frac{\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) + \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \tag{5.13}$$

$$\Delta L = v_0(t_d - t_b) + \left[-v_0\frac{\pi}{2\omega} + \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) + \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \tag{5.14}$$

Subtracting Eqn 5.14 from Eqn 5.13

$$v_0\frac{\pi}{2\omega} - v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = 0$$

$$\frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) = v_0\frac{\pi}{2\omega}\,1 + v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega}$$

$$\frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) = v_0\frac{\pi}{2\omega}\left[1 + \frac{\beta_i V_1}{2V_0}\right] \approx \frac{v_0\pi}{2\omega}$$

$$\sin ce \quad \frac{\beta_i V_1}{2V_0} << 1$$

$$t_d - t_b = \frac{\pi V_0}{w\beta_i V_1} \tag{5.15}$$

From Equation 5.8 and 5.15

$$\Delta L = v_0\frac{\pi V_0}{w\beta_i V_1} \tag{5.16}$$

ΔL is theoretical value of distance from buncher cavity at which bunching of electrons takes place. Refer Fig -5.1

Equation 5.16 gives the design parameter for spacing between buncher and catcher cavities.

However equation – 5.16 is only an approximation, because mutual repulsive force between the electrons in the highly densed beam are not taken into consideration. It will be seen later that optimum spacing between the two cavities "L" optimum is given by (for maximum degree of bunching)

$$L_{optimum} = \frac{3.682 v_0 V_0}{\omega \beta_i V_1}$$

Which is closer to equation – 5.16

(For derivation refer equation – 5.28)

Let T= Transit time for on electron travel distance „L" (function of „t")

L = spacing between two cavities

Let T_0 = Transit time for electron when the field in buncher cavity is i.e. $v(t_1) = v_0$

$$T_0 = \frac{L}{v_0} \tag{5.18}$$

$$T = \frac{L}{v(t_1)}$$

Substituting for $v(t_1)$ from equation 5

$$T = \frac{L}{v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]}$$

$$T = \frac{L}{v_0}\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]^{-1}$$

Using binomial expansion $(1+x)^{-1} = 1-x$ for $x<<1$ and $V_1<<V_0$, $T_0 = L / v_0$

$$T = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \tag{5.19}$$

Multiplying above equation by „ω"

$$\omega T = \omega T_0\left[1 - \frac{\beta_i V_1}{V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \tag{5.20}$$

Let θ_0 = Angular variation in the signal during time „T_0"

$\theta_0 = \omega T_0$

$$\omega T = \omega T_0 - \frac{\omega T_0 \beta_i V_1}{V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

$$\omega T = \theta_0 - X \sin \omega t_1 - \frac{\theta_g}{2}$$

$$where \;\; X = \frac{\beta_i V_1}{2V_0} \theta_0 \qquad (5.21)$$

X is called Bunching parameter of Klystron

The second design criterion is that the maximum energy will be transferred by the electrons to the catcher cavity when the bunch enters the cavity while the field is at negative peak. Assuming the buncher and catcher cavities are at same phase the above condition can be expressed mathematically

$$\theta_0 = \omega T_0 = 2\pi n - \pi/2 = 2\pi N \qquad (5.21A)$$

Where n is an integer and N is number cycles the angle has undergone chages during the transit time T_0

Expression for output current

Let us try to establish the relation between I_0 = dc current passing through buncher vacity and „i_2" ac current in the catcher cavity.

Making use of law of conservation

Let charge „dQ_0" pass through the buncher gap at a time interval „dt_0" and we will assume the same amount of charge passes through the catcher gap later in time interval „dt_2"

$$dQ_0 = I_0 \, dt_0$$

$$dQ_0 = I_0 |dt_0| = i_2 |dt_2| \qquad (5.22)$$

We have earlier defined t_0, t_1, t_2 such that

$\tau = t_1 - t0$

$t_1 = t_0 + \check{T}$

$T = t_2 - t_1$, $\quad T = t_2 - (t_0 + \check{T})$

From equation 19

$$T = t_2 - t_1 = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$t_2 = t_0 + \tau + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

Multiplying by „ω"

$$\omega t_2 = \omega t_0 + \omega\tau + \omega T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$\omega t_2 = \omega t_0 + \theta_g + \theta_0 - \frac{\theta_0\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$we\ have\ X = \frac{\theta_0\beta_i V_1}{V_0}$$

$$\omega t_2 = \omega t_0 + \theta_g + \theta_0 - X\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

Differentially above equation w.r.t. „t_0"

θ_g, θ_0 are constants w.r.t. „t"

$$\omega\frac{dt_2}{dt_0} = \omega - \omega X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right) and$$

$$dt_2 = dt_0\left[1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad (5.23)$$

From equation 22 and 23

$$I_0\left[dt_0\right] = i_2\left|dt_2\right| = i_2\ dt_0\left[1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$i_2\,(t_0) = \frac{I_0}{1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)} \qquad (5.24)$$

i_2, the beam

current at catcher cavity is a periodic waveform ofperiod about dc current I_0 $\frac{2\pi}{\omega} = \frac{1}{f}$,

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0\beta_0 J_n(\times)\cos n\omega(t_2 - \tau - T_0) \qquad (5.25)$$

Where n = integer

Derivations of above equation is out of preview of the syllabus

$J_n(x)$ = n^{th} order bessel function of 1^{st} kind

We are interested in the fundamental component i.e. n=1, ac beam current

Neglecting dc current and higher order of ac current i.e. n>2

I_f = fundamental component of ac current from equation 25 in catcher cavity whose beam complex coefficient = β_0

$$I_f = 2I_0 J_1(X)\beta_0 \cos w\ (t_2 - \tau - T_0) \qquad (5.26)$$

Let I_{fmax}= magnitude of I_f in catcher cavity

$$I_2 = 2\beta_0 I_0 J_1(X) \qquad (5.27)$$

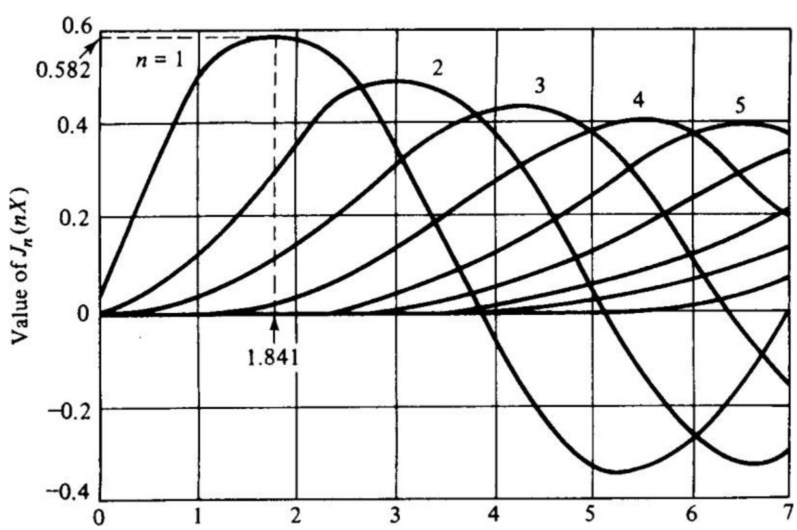


Fig 5.4: Bessel function J_n (nX)

$J_1(X)$ is maximum at X = 1.841 i.e. $J_1(1.841)$ = 0.582 from Bessel function

Where X = Bunching parameter of Klystron as defined in equation 21

$$X = \frac{\beta_i V_1}{2V_0}\theta_0 = \frac{\beta_i V_1}{2V_0} w \frac{L}{v_0}$$

$L \rightarrow L_{optimum}$ as $X \rightarrow 1.841$

$$Loptimum = \frac{2 \times 1.841 \times V_0 \times \nu_0}{\beta_i V_1 \omega} = \frac{3.682 \nu_0 V_0}{\omega \beta_i V_1} \tag{5.28}$$

The same equation is given theoretically as equation 5.17 earlier

<u>The Output power and beam loading</u>

Let I_2 is max value of current in the catcher cavity with $\beta=\beta_0$

From equation 5.27

$$I_2 = 2\beta_0 I_0 J_1(X) \tag{5.29}$$

<u>Power Output and efficiency of Klystron</u>

The equivalent output circuit of Klystron is

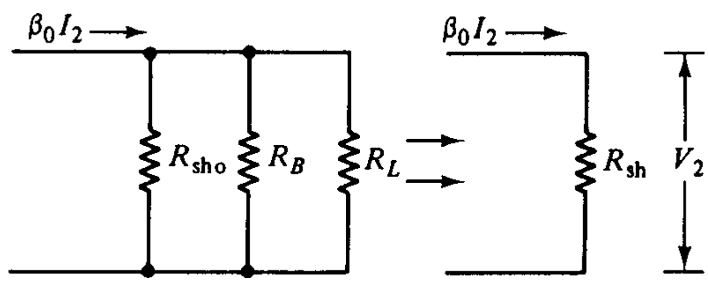


Fig 5.5 : Output equivalent circuit

Where R_{sho} = Wall resistance of catcher cavity
R_B = beam loading resistance
R_L = External load resistance
R_{sh} = Effective shunt resistance = $R_{sho} \| R_B \| R_L$

$$i_{rms} = \frac{i_{f\,max}}{\sqrt{2}} = \frac{I_2}{\sqrt{2}} = \frac{2I_0\beta_0 J_1(X)}{\sqrt{2}}$$

$$Poutput = i^2_{rms}\ Rsh = 2I_0^{\,2}\beta_0^{\,2} J_1^{\,2}(X) R_{sh} = \beta_0 I_0 J_1(X) V_2 \tag{5.29A}$$

$$\max imum\ output\ \ voltage = V_2 = i_{f\,rms}\ Rsh = 2\beta_0 I_0 J_1(X) R_{sh} \tag{5.30}$$

$$Efficiency = \eta = \frac{P_0}{p_{dc}} = \frac{2I_0^{\,2}\beta_0^{\,2} J_1^{\,2}(X) R_{sh}}{V_0 I_0}$$

$$\eta = \frac{\beta_0 I_0 J_1(X) V_2}{V_0 I_0} = \beta_0 J_1(X)\frac{V_2}{V_0} \qquad (5.31)$$

Maximum theoretical efficiency of Klystron is

$\beta_0 = 1$, $J_1(X) = 0.582$, $V_2 = V_0$

η_{max} = 58.2 % theoretical

Practically $\eta \approx 40\%$

Condition for maximum transfer of energy to catcher cavity

From equation 5.21A we have $\theta_0 = \omega T_0 = 2\pi n - \pi/2 = 2\pi N$ and from equation 5.21

$$X = \frac{\beta_i V_1}{2V_0}\theta_0 = \frac{\beta_i V_1}{2V_0} 2\pi N$$

$$\left(\frac{V_1}{V_0}\right)_{max} = \frac{2 \times 1.841}{2\pi N} = \frac{1.841}{\pi N} \qquad (5.32)$$

Mutual Conductance 'G_m'

$$G_m = \frac{I_2}{V_1} = \frac{2 I_0 \beta_i J_{1(X)}}{V_1} \qquad (5.33)$$

From equation 5.21 we have

$$X = \frac{\beta_i V_1 \theta_o}{2V_o} = \frac{\beta_i V_1}{2V_o}\frac{\omega L}{v_o}$$

$$V_1 = \frac{2 v_0 V_o X}{\omega \beta_i L} \qquad (5.34)$$

Substituting V_1 in equation 5.33

$$G_m = \frac{2 I_0 \beta_o J_{1(X)}}{2 v_o V_o X}\omega \beta_i L$$

Let $\beta_i = \beta_o = \beta$

$$G_m = \frac{\beta^2 \omega L J_{1(X)}}{v_o X}\frac{I_0}{V_o} \qquad (5.35)$$

Let R_o be the dc beam resistance of buncher cavity.

Let G_o be the dc beam conductance of buncher cavity.

$$G_o = \frac{I_0}{V_o} \quad \text{and} \quad R_o = \frac{V_0}{I_0}$$

$$G_m = \frac{\beta^2 \omega\, L\ \ J_{1(X)}}{v_o\, X}\ Go \qquad (5.36)$$

$$\frac{Gm}{Go} = \frac{\beta^2 \omega\, L}{v_o}\ \frac{J_{1(X)}}{X} \qquad (5.37)$$

The maximum value of $\frac{Gm}{Go}$ is obtained by J_1 (X) = 0.582 for X - 1.841 and β = 1

$$\left[\frac{Gm}{Go}\right]_{max} = \frac{0.316\omega\ L}{v_o} \qquad (5.38)$$

Voltage gain A_v:

$A_v = \frac{V_2}{V_1}$ substituting for V_2 from equation 5.30

$$A_v = \frac{V_2}{V_1} = \frac{2\ \beta_o\ \ I_o\, J_{1(X)}\, R_{sh}}{V_1}$$

substituting for V_1 from equation 5.34, and taking $\beta_i = \beta_o = \beta$

$$A_v = \frac{\beta_o\ \ I_o\, J_{1(X)}\, R_{sh}}{v_o\ \ V_o\, X}\ \omega\, \beta_i\ L \qquad (5.39)$$

Since $\frac{\omega \beta_i L}{v_o} = \theta_o$

$$A_v = \frac{\beta^2 \theta_o\ \ J_{1(X)}}{X}\ \frac{I_o}{V_o}\ R_{sh} \qquad (5.40)$$

Substituting for G_m from equation 5.35

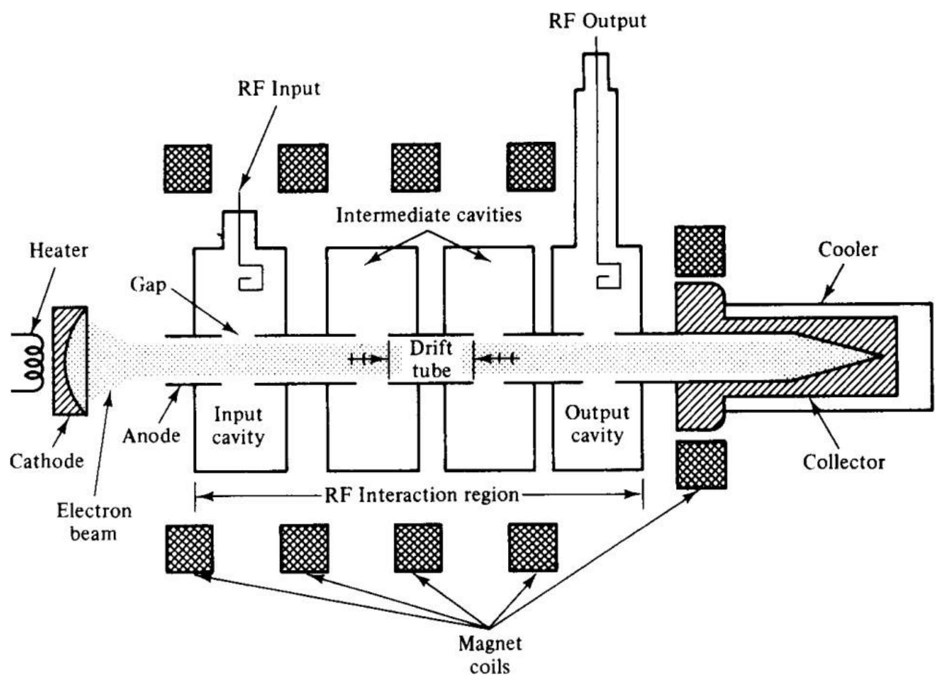
$$A_v = G_m\, R_{sh} \qquad (5.41)$$

Multi Cavity Klystron

- Typical gain of 2-cavity klystron is 30 dB. This gain is not adequate in many applications

- In order to achieve higher gain, several two cavity resonant tubes are connected in cascade in which output of each of the tubes is fed as input to the following tube.

- The intermediate cavities are place at a distance so that the bunching parameter X = 1.841 with respect to the previous cavity.
- The intermediate cavity acts as a buncher with the passing electron beam inducing a more enhanced RF voltage than the previous cavity, which in turn sets up an increased velocity modulation.
- Typical gain achievable by a multi cavity klystron is of the order of 50 dB with bandwidth of about 80 M Hz.
- A multi cavity klystron amplifier produces high gain and narrow bandwidth if all the cavities are tuned to the same frequency.
- When each of the cavities are tuned to slightly different frequencies (Staggered tuning), the bandwidth will appreciably increase but at the cost of the gain.



Schematic diagram of a 4 –cavity klystron

Beam current density in klystrons and plasma frequency

While carrying out the mathematical analysis of 2-cavity klystron earlier, the space charge effect (mutual repulsive forces between the electrons) was neglected. This is acceptable for a low power amplifier with small density of electrons in the beam.

However, when high power klystron tubes are analysed, the electron density in the beam is large and forces of mutual repulsion of electrons cannot be neglected.

When the electrons perturbate (oscillate) in the electron beam, the electron density consists of a dc part t RF puturbation caused by the electron bunches. The space change forces within electron bunch vary with shape and size of an electron beam.

Mathematically, the charge density and velocities of perturbations are given bye

Charge density = ρ = - B cos (β_ez) cos (w_qt+ θ) (5.42)

Velocity perturbation = v = - C sin (β_ez) sin (w_qt+ θ) (5.43)

Where

B = constant of charge perturbation
C= constant of velocity perturbation
$\beta_e = \omega / v_0$ = is the dc – phase constant of electron beam
ω_p = Plasma frequency

$$\omega_p = \sqrt{\frac{e\rho_0}{\varepsilon_0 \ m}} \quad (5.44)$$

$\omega_q = R.\ \omega_p$ is the reduced plasma frequency
θ = phase angle of oscillation
ρ_0 = dc electron charge density
R = Reduction factor
v_0= dc electron velocity
ρ = Instantaneous RF charge density

2- cavity klystron as on oscillator

- An amplifier can be used as on oscillator with appropriate feedback
- The 2-cavity klystron oscillator is obtained by simply providing a feedback loop between the simply providing a feedback loop between the input and output cavities of the klystron.
- The condition for sustained oscillations is

$$\theta + \propto + \frac{\pi}{2} = 2\pi n \ \ rad \quad (5.45)$$

Where
θ = Total phase shift in the resonators and the feedback cable
$\propto + \frac{\pi}{2} = phase\ \ angle\ \ between\ \ buncher\ \ and\ \ catcher\ \ voltages$

n = an integer

If the two resonators oscillate in phase i.e. θ = 0

then $\propto = 2\pi n - \frac{\pi}{2} =$ *for* max *power output*

- If the resonators are de-funed, the oscillations can be obtained over a wide range of frequencies
- The frequency stability of oscillator is obtained by
  (a) controlling the temperature of resonation
  (b) Use of regulated power supplies

Solved problems on 2-cavity Klystron

eg1.The parameters of a two cavity Klystron are: (May 2009)

Input power = 10 mW
Voltage gain = 20 dB
R_{sh}(Input cavity) = 25 kΩ
R_{sh}(Output cavity) = 35 kΩ
Load resistance = 40 kΩ
<u>*Calculate*</u> *(a) Input voltage (b) Output voltage (c) Power output*

<u>Solution</u>

From input ac equivalent circuit

$P_{ac\ in} = \frac{V1^2}{Rshin}$

$V_1 = \sqrt{Pacin * Rshin} = \sqrt{250} = 15.81$ V

$A_v = 20 \log \frac{V2}{V_1} = 20$ dB

$\log \frac{V_2}{V_1} = 1, \quad \frac{V_2}{V_1} = 10, \quad V_2 = 158.1$ V

Power output $= \frac{V_2^2}{RL \,||R_{shout}} = 1.339$ W

eg2: A 2-cavity Klytron amplifier has following parameters (May 2007)

V_o = 1200 V
I_o = 25 mA
R_o = 30 kΩ
f = 10 GHz

d = 1 mm
L = 4 cm
Rsh = 30 kΩ

Calculate (a) Input voltage for max output (b) Voltage gain (c) Klystron efficiency

Solution

$$L_{optimum} = \frac{3.682 v_0 V_0}{\omega \beta_i V_1}$$

$$\therefore V_1 = \frac{3.682\ v_o V_o}{\omega \beta_i\ L}$$

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} = 0.593 \times 10^6 \times \sqrt{1200}$$ *= 0.2054 X 10^8 m/s*

$$\theta_g = \frac{\omega\ d}{v_0} = \frac{2\pi \times 10 \times 10^9 \times 10^{-3}}{2.054 \times 10^7} = 3.058 rad$$

$$\beta_i = \frac{\sin \theta_g / 2}{\theta_g / 2} = \frac{\sin 1.529}{1.529} = 0.653$$

$$V_1 = \frac{3.682\ v_o V_o}{\omega \beta_i\ L}$$ X = **55.3 V**

$$V_2 = 2\beta_0 I_0 J_1(X) R_{sh}$$

$= 2 \times 0.653 \times 25 \times 10^{-3} \times 0.582 \times 30 \times 10^3$ = **570.069 V**

$A_v = 20 \log \frac{V2}{V1}$ = **20.26 dB**

$$\eta = \beta_0 J_1(X) \frac{V_2}{V_0} =$$

$$\frac{0.653\ X\ 0.582\ X\ 570.069}{1200} = 0.1805 \quad \text{or } 18.05\%$$

eg 3: A 2-cavity klystron amplifier is tuned at 3 G Hz. The drift space length is 2 cm and beam current is 25 mA. The Catcher voltage is 0.3 times the beam voltage and β = 1. Calculate

(a) Power output and efficiency for N = 5.25
(b) Beam voltage, input voltage and output current for N=5.25

Solution

$$\Theta_o = \frac{\omega\ L}{v_o} = 2\pi N$$

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} = \frac{\omega\ L}{2\pi N}$$

$$V_0 = \left[\frac{\omega\ L}{\pi N \text{ X } 0.593 \text{ X } 10^6}\right]^2 = 371.4 \text{ V}$$

$V_2 = 0.3\,V_0 = 111.4$ V

From Eqn 5.32 we have

$$\left(\frac{V_1}{V_0}\right)_{max} = \frac{1.841}{\pi N} = 0.11$$

$V_{1\ max} = 0.11 \times 371.4 = 41.46$ V

From equation 5.29 $Poutput = \beta_0 I_0 J_1(X) V_2$ = 25 X 10-3 X 0.582 X 111.4 = 1.62 W

$$\eta = \frac{P_{out}}{V_0 I_0} = \frac{1.62L}{371.4 \text{ X } 25 \text{ X } 10-3} = 0.174 \text{ or } 17.4\%$$

eg4: *A 2-cavity klystron operates at 5 GHz with a dc voltage of 10 kV and a 2 mm cavity gap. For a given RF voltage the magnitude of gap voltage is 100 volts Calculate*

(a) Transit angle (b) Velocity of electron leaving the gap

$$v_0 = 0.593 \times 10^6 \sqrt{10000} = 0.593 \text{ X } 10^8 \text{ m/s}$$

$$\tau = \frac{d}{v_o} = 33.7 \text{ X } 10^{-12} \text{ s}$$

$$\theta_g = \frac{\omega\ d}{v_0} = \frac{2\pi \times 5 \times 10^9 \times 2x\ 10^{-3}}{v_0} = 1.058 \quad rad$$

$$\beta_i = \frac{\sin\theta_g/2}{\theta_g/2} = 0.954$$

From equation 5.11 and 5.12 we have

$$v_{max} = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right) \quad 0.5 \left(1 + \frac{0.954x100}{}\right) \quad 108$$

$2x10000$ $\bar{=}$ 0.5986 x 10 m/s

IIIly $v_{min} = v(t_1) = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right) = 0.593 \text{X} 108 \left(1 - \frac{0.954x100}{2x10000}\right) = 0.59 \times 10^8$ m/s

eg 5: *: A four cavity klystron amplifier has the following parameters*

$V_0 = 18$ kV , $I_0 = 2.25$ A , $d = 1$ cm
$f = 10$GHz, $V_1 = 10$V (rms) , $\beta_i = \beta_0 = 1$
ρ_0 = dc electron beam density = 10^{-8} C/ m^3
Determine

1. *dc electron velocity 'υ_0'*
2. *dc electron phase constant 'β_e'*
3. *plasma frequency 'ω_p'*
4. *reduced plasma frequency 'ω_q' for R = 0.5*
5. *Reduced plasma phase constant 'β_q'*
6. *Transit time across input gap*

Solution

(a) The dc electron velocity is

$$\upsilon_0 = 0.593 \times 10^6 \sqrt{18{,}000} = 0.796 \times 10^8 \, m/s$$

(b) The dc electron phase constant is

$$\beta_e = \frac{\omega}{\upsilon_0} = \frac{2\pi \times 10 \times 10^9}{0.796 \times 10^8} = 7.89 \times 10^2 \, rad/m$$

(c) The plasma frequency

$$\omega_p = \sqrt{\frac{e\rho_0}{\varepsilon_0 m}} = \left[\frac{1.759 \times 10^{11} \times 10^{-8}}{8.854 \times 10^{-12}}\right]^{1/2} = 1.41 \times 10^7 \, rad/s$$

(d) The reduced plasma frequency

ω_q = R. ω_p = 0.5 x 1.41 x 10^7 = 0.705 x 10^7 rad / s

(e) The reduced plasma phase constant

$$\beta_q = \frac{\omega_q}{\upsilon_0} = \frac{0.705 \times 10^7}{0.796 \times 10^8} = 0.088 \;\; rad/m$$

(f) Transit time across the gap

$$\tau = \frac{d}{\upsilon_0} = \frac{10^{-2}}{0.796 \times 10^8} = 0.1256 \;\; ns$$

Eg 6: *A 2-cavity klystron operates at 10 GHz with*

V_0 = 10 kV , I_0 = 3.6 mA , L= 2 cm

G_{shout} = 20 μ mhos, β = o.92, R_{shin} = 80 k ohms

Findout

(a) Max voltage gain (b) Max Power gain

Solution

$$\upsilon_0 = 0.593\times10^6\sqrt{10{,}000} = 0.593\times10^8\, m/s$$

$$\theta o = \frac{\omega\ L}{v_0} = \frac{2\pi\times10\times10^9\times2x\ 10^{-2}}{v_0} = 21.187\quad rad$$

R_{shout} = 1/ G_{shout} = 50 k Ohms

For Max A_v, J1(X) = 0.582 and x = 1.841, From equation 5.40

$$\mathbf{A_v} = \frac{\beta^2\theta_o\ J_{1(X)}}{X}\frac{I_0}{V_o}R_{sh} = \frac{.92\ x\ .92\ x\ 21.187\ x\ .582\ x\ 3.6\ x\ 50}{18410} = 1.020$$

Power gain = $A_v^2 R_{shin}/R_{shout}$ = $\frac{\mathbf{1.02\ x\ 1.02\ x\ 80}}{\mathbf{50}}$ = **1.664**

Eg 7 *: 2-cavity klystron operates at 4 GHz with V_0 = 1 kV , I_0 = 22 mA , d= 1 mm, R_{shout} = 20 μ mhos, L = 3 cm . If the dc beam conductance and catcher cavity total equivalent conductance are 0.25 x 10^{-4} mhos and 0.3x 10^{-4} mhos respectively*

Findout

(a) Beam coupling coefficient

(b) dc transit angle in the drift space

© Input cavity voltage V_1 for max V_2

(d) Voltge gain and efficiency (neglecting beam loading)

Solution

$$\upsilon_0 = 0.593\times10^6\sqrt{1000} = 0.188\times10^8\, m/s$$

$$\theta_g = \frac{\omega\ d}{v} = \frac{2\pi\times4\times10^9\times1x}{}$$

v $10-3$

$= \qquad rad$

1.336

$0 \qquad 0$

$$\theta o = \frac{\omega \ L}{v_0} = \frac{2\pi \times 4 \times 10^9 \times 3x \ 10^{-2}}{v_0} = 40.1 \quad rad$$

$V_1 = \frac{3.682\ v_o V_o}{\omega \beta_i\ L} X$ = **99.05 V** considering X= 1.841

From equation 5.41

A_v $= \frac{\beta^2 \theta_o \ J_{1(X)}}{X} \frac{I_O}{V_o} R_{sh} = \frac{\beta^2 \theta_o \ J_{1(X)}}{X\ G_{sh}} \frac{I_O}{V_o} = 7.988$

$V_2 = A_v V_1 = 7.988 \times 99.05 = 791.2$ V

From equation 5.31

$$\eta = \beta_0 J_1(X) \frac{V_2}{V_0}$$ Considering $J_1(X) = 0.582$

= 0.4268

REFLEX KLYSTRON

Reflex klystron is a single cavity law power microwave oscillator. The characteristics of Reflex Klystron are

Power output: 10- 500mw

Frequency range: 1 to 25 GHz

Efficiency: 10-20%

Applications

1. Widely used in the as a source for microwave experiments
2. Local oscillator in microwave receivers

The theory of the 2-cavity klystron can be applied to the analysis of Reflex klystron with slight modifications

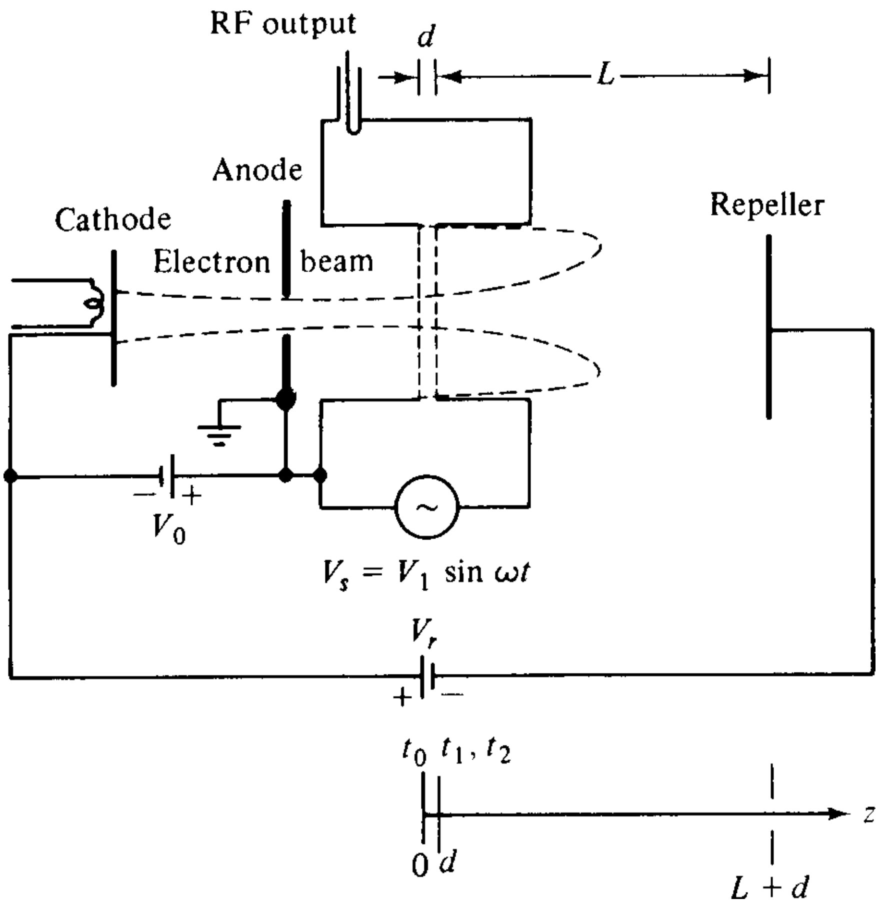


Fig 5.6: Schematic digram of Reflex Klystron

1. Cathode
2. Anode grid
3. Cavity resonator at potential of $+v_0$ w.r.t cathode
4. Repeller at potential of - V_r w.r.t. cathode

Formation of electron beam with uniform velocity v_0 up to civility resonator is similar to that of 2-cavity klystron

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \ m/s$$

Due to dc voltage in the cavity circuit, RF noise is generated in the cavity. This em noise field in the cavity get pronounced at cavity resonant frequency and acts as a small signal microwave voltage source of V_1 sin wt.

The electron beam with uniform velocity v_0 when enters the cavity undergoes velocity modulation as in the case of 2-cavity klystron.

Let t_0 = time at which electron enters the cavity gap

t_1 = time at which electron leave the cavity gap

d = cavity gap

Z = Axis as shown in schematic diagram

Z = 0 at the input gap of cavity

Z = d at the output gap of cavity

Z = L at the reseller

From equation 5 of 2 cavity klystron

$$\nu(t_1) = \nu_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(wt_1 - \frac{\theta_g}{2} \right) \right]$$

Some electrons are accelerated by the accelerating field (during +ve cycle of RF field) and enter the repeller space with greater velocity compared to the electrons with unchanged velocity, some electrons are decelerated by the decelerating field (during -ve cycle of RF field) and enter repeller space with less velocity

All the electrons entering repeller space are retarded by the repeller which is at a -ve potential of $-v_r$. All the electrons are turned back and again enter the cavity in a bunched manner. The bunch re enter the cavity and when field in the cavity is a retarding field bunches convey kinetic energy to the cavity. The cavity converts this kinetic energy into electron magnetic energy at the resonant frequency resulting in the sustained oscillations and therefore the output of the cavity is $V_1 \sin \omega t$

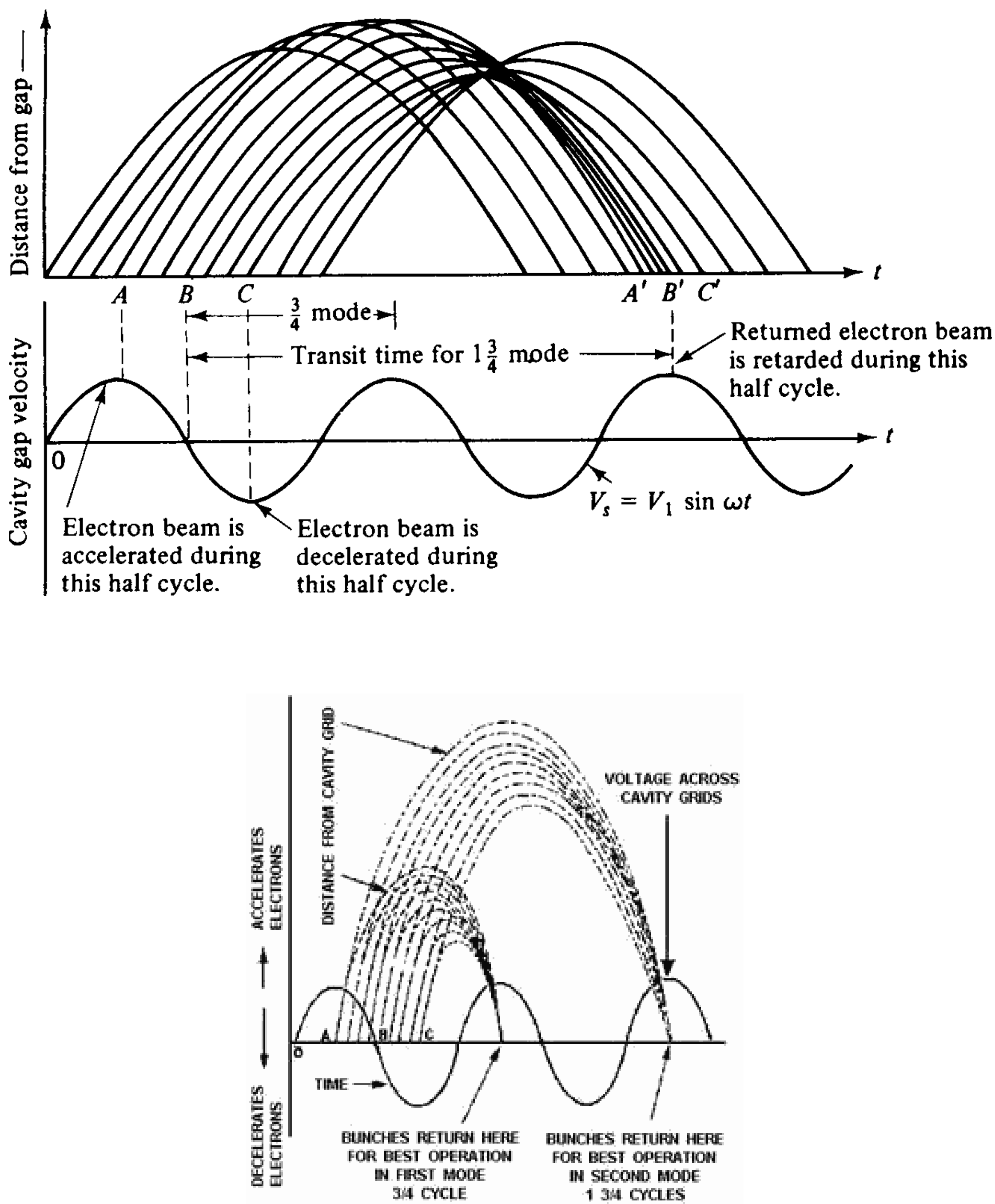


Fig 5.7: Applegate diagram of Reflex Klystron

Let „b" be the reference electron at t = t_2 for our analysis. Electron „b" is passing through the cavity gap while the field is zero (-ve shape) when the electrons a,b,c… leave the cavity i.e. at z = d, the velocity is given by equation

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(wt_1 - \frac{\theta_g}{2}\right)\right] \tag{5.46}$$

These electrons are subjected to retarding field due to repeller voltage during the drift space from z = d to z = L. the retarding field in the drift space is given by

$$E = \frac{V_r + V_0 + V_1 \sin wt}{L} \tag{5.47}$$

The force equation for an electron in the repeller region is given by

$$m\frac{d^2z}{dt^2} = -eE = -e\frac{[V_r + V_0 + V_1 \sin wt]}{L}$$

Since $V_1 << V_0$, $V_1 << V_r$

$$m\frac{d^2z}{dt^2} = -e\frac{(V_r + V_0)}{L} \tag{5.48}$$

Integrating the above equation

$$\frac{dz}{dt} = \frac{-e(V_r + V_0)}{mL}\int_{t_1}^{t} dt = \frac{-e(V_r + V_0)}{mL}(t - t_1) + K_1$$

Where

K_1 = integration constant

t_0 = Time at the electron enters the gap

t_1 = Time at the electron leave the gap

t_2 = Time at the electron re-enters the gap due to retarding field

at t = t_1, z = d, $v(t_1) = dz / dt$

$K_1 = dz/ dt = v(t_1)$

Integrating the above equation once again

$$z = \frac{-e(V_0 + V_r)}{mL}\int_{t_1}^{t}(t - t_1)dt + \int_{t_1}^{t} v(t_1)dt$$

$$z = -e\frac{(V_0 + V_r)}{2mL}(t - t_1)^2 + v(t_1)(t - t_1) + K_2$$

At t= t_1, z= d

K_2 = d

$$z = -e\frac{(V_0 + V_r)}{2mL}(t - t_1)^2 + v(t_1)(t - t_1) + d \qquad (5.49)$$

At t = t_2 electrons returns of cavity after retardation at t = t_2, z=d substituting this in above equation.

$$d = \frac{-e(V_0 + V_r)}{2mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1) + d$$

$$o = \frac{-e(V_0 + V_r)}{2mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1)$$

Let T be rannd trip transit time = t_2-t_1

$$O = (t_2 - t_1)\left[\frac{-e(V_0 + V_r)}{2mL}(t_2 - t_1)^2 + v(t_1)\right]$$

$$\frac{e\,(V_0 + V_r)}{2mL}(t_2 - t_1) = v(t_1) \qquad (5.50)$$

$$T' = t_2 - t_1 = \frac{2mL}{e(V_0 + V_r)}v(t_1)$$

$$= T_0'\left[1 + \frac{\beta_i V_i}{2V_0}\sin wt_1 - \frac{\theta_g}{2}\right]$$

$$where\ \ T_0' = \frac{(2mL)}{e\,V_0 + V_r}v_0 \qquad (5.51)$$

T_O"Is the round trip transit time of electron „b" which is learning the cavity at velocity $v(t_1) = v_0$

T"$_0$ is a function of V_r

$\omega(t_2-t_1) = \omega T$

$$\theta' = \omega T' = \omega T'_0 + \omega T'_0 \, \frac{\beta_i V_i}{2V_0} \sin\left(wt_1 - \frac{\theta_g}{2} \right)$$

Let $\omega T_0 = \theta_0$. From equation 5.21 we have

$$\theta' = \theta'_0 + X'\sin\left(wt_1 - \frac{\theta_g}{2} \right) \quad (5.52)$$

Where X" is bunching parameter of Reflex Klystron

Power output and efficiency of Reflex Klystron

In case of 2-cavity klystron, we had seen that the maximum transfer of kinetic energy to the cavity takes place when the electron bunch enters when the field is -ve peak

Similarly in the case of reflex Klystron, the bunch must enter cavity when the field is +ve peak. (This is because the direction of electron bunch entering into the cavity is 180^0 opposite to that of 2-cavity Klystron)

Considering the above condition we can see from the applegate diagram that the round trip transit time of reference electron is

$$\omega(t_2 - t_1) = \omega T_0 = \left(n - \frac{1}{4} \right) 2\pi = 2\pi n - \frac{\pi}{2} = 2\pi N \quad (5.53)$$

When n = 1,2,3……………….

Let N= n-1/4 is called the mode number

Therefore $\theta_0 = 2\pi N$

Applying the same analogy of 2-cavity klystron and using equation 25, the current in the cavity can be expressed as

$$i_2 = -I_0 - \sum_{n=1}^{\infty} 2I_0 J'_n (n\times') \cos\left[\pi (\omega t_2 - \theta_0 - \theta_g \right] \quad (5.54)$$

The fundamental component of current in the cavity if at n =1 is

$$i_f = -\beta_i i_2 \Big|_{n=1} = 2I_0 \beta_i J_1 (\times^1) \cos(\omega t_2 - \theta_0) \quad (5.55)$$

$$\theta_g << \theta_0$$

Maximum magnitude of fundamental component current in the cavity I_2

$$I_2 == 2I_0 \beta_i J_1 (\times^1)$$

V_2 = output voltage of the cavity = V_1 (except for the phase difference)

$$V_{sh}^{\ 1} = \qquad V_2$$

$$= 2I_0 \qquad \beta_i J_1 \quad (\times^1) \quad R \qquad (5.57)$$

$$output\ \ power = P_{az} = i^2_{rms}.R_{sh} = \left(\frac{2I_0\beta_i J_1(\times^1)}{\sqrt{2}}\right)^2 R_{sh} \tag{5.58}$$

$$P_{ac} = 2I^2{}_0\beta^2{}_i J_1^{\ 2}(\times^1)R_{sh} \tag{5.59}$$

$$P_{ac} = V_1.I_0\beta_i J_1(\times^1) \tag{5.60}$$

We have earlier seen that

$$X^1 = \frac{\beta_i V_1}{2V_0}\theta_0 = \frac{\beta_i V_1}{2V_0}\left(2\pi n - \frac{\pi}{2}\right)$$

$$\frac{V_1}{V_0} = \frac{2X^1}{\beta_i\left(2\pi n - \frac{\pi}{2}\right)} \tag{5.61}$$

$$\frac{P_{ac}}{P_{dc}} = Power\ \ Efficiency = \eta = \frac{V_1 I_0 \beta_i J_1(\times^1)}{V_0 I_0} \tag{5.62}$$

$$= \frac{V_1}{V_0}.\frac{\beta_i J_1(\times^1)}{I_0} = \frac{2X^1 J_1(\times^1)}{\left(2\pi n - \frac{\pi}{2}\right)}$$

$$\eta = \frac{2X^1 J_1(\times^1)}{\left(2\pi n - \frac{\pi}{2}\right)} \tag{5.63}$$

The product $X_1J_1(X^1)$ is maximum at $X^1 = 2.408$, $J_1(X^1) = 0.52$

$X^1J_1(X^1)_{max}=1.25$ at $X^1 = 2.408$

$$\eta_{max} = \frac{2 \times 1.25}{\left(2\pi n - \frac{\pi}{2}\right)}$$

At n = 2 (n=1 too short a value)

$\boldsymbol{\eta_{max}}$ = 0.227 or 22.7 (5.64)

From equation 5. 50 we have

$$T'_0 = \frac{2mLv_0}{e(V_0 + V_r)} \quad \text{where } v_0 = \sqrt{\frac{2e}{m}V_0}$$

From equation 5.53, $\omega T''_0 = 2\pi n - \pi/2$

$$\omega T_0 = \frac{2\omega m L v_0}{e(V_0 + V_r)} = \frac{2\omega m L}{e} \cdot \sqrt{\frac{2e}{m}} \frac{\sqrt{V_0}}{V_0 + V_r} = 2\pi n - \frac{\pi}{2} \quad (5.65)$$

$$\frac{V_0}{(V_0 + V_r)^2} = \frac{\left(2\pi n - \frac{\pi}{2}\right)^2}{8\omega^2 L^2} \cdot \frac{e}{m} \quad (5.66)$$

The above equation gives relationship between V_0, V_r and „n" for given V_0, $n = f(V_r)$

From equation 5. 60 we have $V_1 = \dfrac{2X^1 V_0}{\beta_i \left(2\pi n - \frac{\pi}{2}\right)}$

From equation 5.58, $P_{ac} = V_1 I_0 \beta_i J_1(X^1)$

$$P_{ac} = \frac{2X^1 V_0 I_0 \beta_i J_1(\times^1)}{\beta_i \left(2\pi n - \frac{\pi}{2}\right)} = \frac{2V_0 I_0 X^1 J_1(\times^1)}{2\pi n - \frac{\pi}{2}} \quad (5.67)$$

Substituting for $2\pi n - \pi/2$ from equation 5.65

$$= \frac{2V_0 I_0 X^1 J_1(\times^1)}{2\omega m L} \sqrt{\frac{m}{2e}} \cdot \frac{e(V_0 + V_r)}{V_0}$$

$$power\ output = P_{ac} = \frac{V_0 I_0 X^1 J_1(\times^1)(V_0 + V_r)}{\omega L} \frac{\sqrt{e}}{\sqrt{2mV_0}} \quad (5.68)$$

Equation 5.68 gives relationship between relationship between

1. V_r and P_{ac}
2. ω and P_{ac}

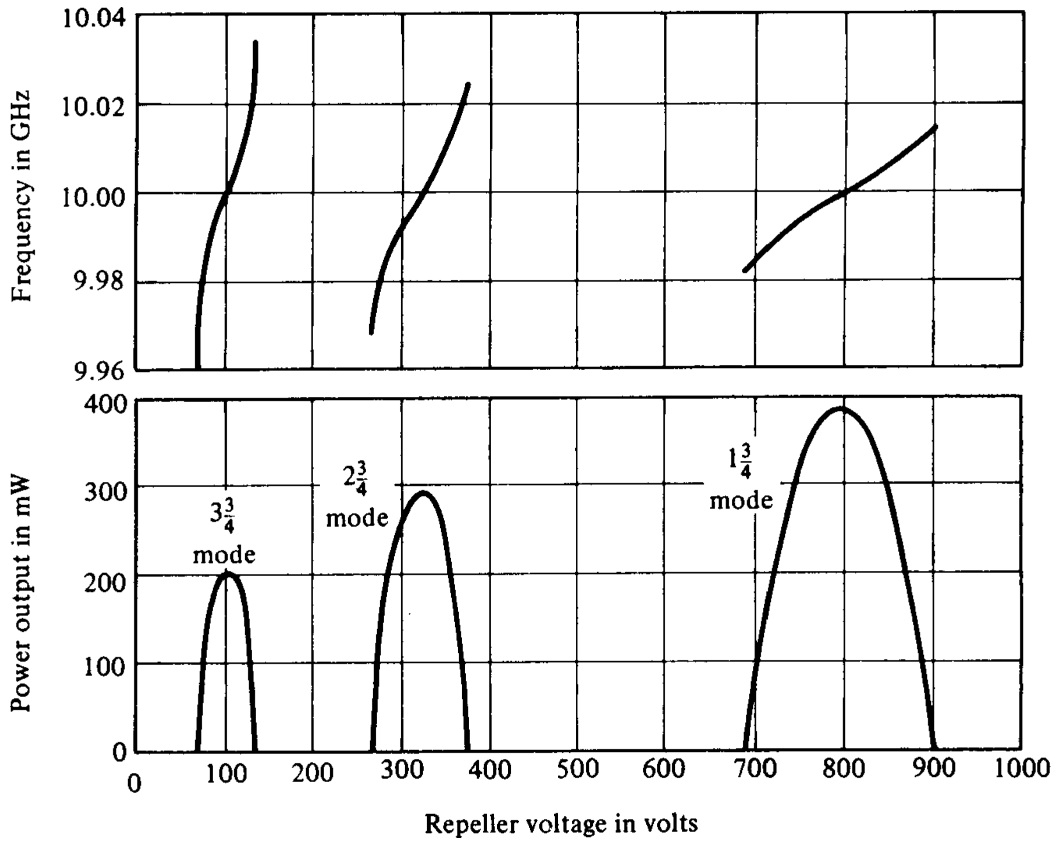


Fig 5.8: Power output and frequency characteristics of reflex klystron

Power output frequency as a function of V_r for various modes

The above fig gives the characteristic of Reflex Klystron

Electronic Admittance of Reflex Klystron

We have seen earlier that the fundamental component of current in the cavity is

$$I_f = 2I_0\beta_i J_1(\times^1)\cos(\omega t_2 - \theta_0)$$

The above equation in the phasor form is

$$I_f = 2I_0\beta_i J_1(\times^1)e^{-j\theta_0} \quad (5.69)$$

There is a phase difference of п/2 between the voltages V_1at t= t_0 and V_2 at t= t_2

$$V_2=V_1^{-ej\pi/2} \quad (5.70)$$

The electronic admittance Y_e is defined as the ratio of i_f to V_2

$$Ye = i_f / V_2 \quad (5.71)$$

From equation 5.69, 5.70 and 5.71

$$Y_e = \frac{I_0}{V_0}\beta_i^2\theta'_0 \cdot \frac{J_1(X')}{X'} e^{j\left(\frac{\pi}{2}-\theta_0\right)}$$

$where\ Y_0 = \frac{I_0}{V_0}$ =dc beam admittance

$$Y_e = Y_0 \beta_i^2\theta'_0 \cdot \frac{J_1(X')}{X'} e^{j\left(\frac{\pi}{2}-\theta_0\right)} \qquad (5.72)$$

Ye is a complex quantity and can be written as $Y_e = G_e + j B_e$

It can be seen that the amplitude of electronic admittance is proportional to dc beam admittance and also transit angle or N

The equivalent circuit of Reflex Klystron is given by

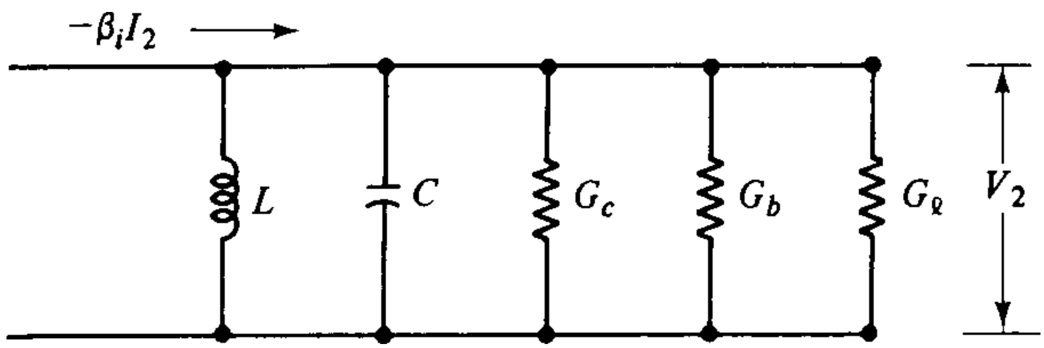


Fig 5.9: Equivalent circuit of reflex klystron

Where G_c = copper loss conductance

G_b = Beam loading conductance

G_L = Load conductance

L and C are energy storage elements of the cavity

G = Conductance of the cavity including external load =

$G = G_c + G_b + G_L = 1 / R_{sh}$ (5.73)

Equation 5.69 can also represented as

$Y_e = G_e + jB_e$ (5.74)

Where G_e = Electronic conductance

B_e = Electronic Sustenance

The necessary condition for oscillations is that the magnitude of the real part of the admittance „G_e" should not be less than total conductivity of the cavity „G"

$$|-G_e| \geq G \qquad (5.75)$$

This condition is pictorially represented in the figure below

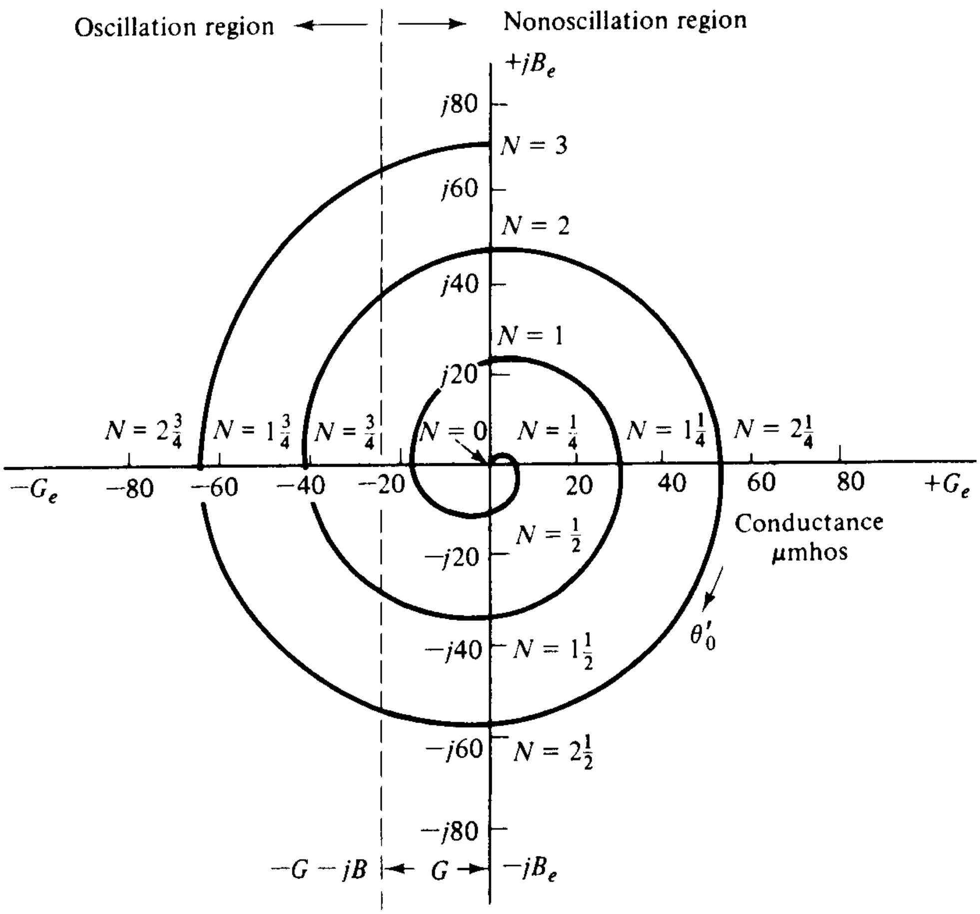


Fig 5.10: Electronic Admittance spiral of reflex klystron

Electronic and mechanical tuning of Reflex Klystron

For using the device as varying frequency oscillator

Electronic tuning is possible by adjustment of repeller voltage „V_r". the tuning range is about ± 8 MHz in X – band. For higher bands tuning to an extent of ±80 MHz is practicable.

From equation 6.66, we have

$$(V_0 + V_r)^2 = \frac{8mL^2V_0}{\left(2\pi n - \frac{\pi}{2}\right)^2 .e}\omega^2 \tag{5.76}$$

Differentiating V_r w.r.t ω

$$2(V_r + V_0)\frac{dV_r}{d\omega} = \frac{16mL^2V_0}{\left(2\pi n - \frac{\pi}{2}\right)^2 .e}\omega$$

$$\frac{dV_r}{d\omega} . = \frac{8mL^2V_0}{e\left(2\pi n - \frac{\pi}{2}\right)^2}\frac{1}{V_r + V_0}$$

Substituting for $V_r + V_0$ from equation 5.76 above

$$\frac{dV_r}{d\omega} = \frac{8mL^2V_0\,\omega}{e\left(2\pi n - \frac{\pi}{2}\right)^2}\frac{\left(2\pi n - \frac{\pi}{2}\right)}{2L\omega}\sqrt{\frac{e}{2mV_0}}$$

$$\frac{dV_r}{d\omega} = \frac{4L}{2\pi n - \frac{\pi}{2}}.\frac{V_0 m}{2e} = \frac{L}{2\pi n - \frac{\pi}{2}}\sqrt{\frac{8mV_0}{e}}$$

$$\omega = 2\pi f$$

$$\frac{dV_r}{df} = \frac{2\pi L}{2\pi n - \frac{\pi}{2}}\sqrt{\frac{8mV_0}{e}} \tag{5.77}$$

$$df = \frac{dV_r}{\frac{2\pi L}{2\pi n - \frac{\pi}{2}}\sqrt{\frac{8mV_0}{e}}} \tag{5.78}$$

Equation 5.78 gives the relation between variation V_r and the resulting variation in the frequency.

Mechanical Tuning of Reflex Klystron

The resonant frequency of the cavity can be adjusted using following two methods

1. The frequency of resonance is mechanically adjusted by adjustable screws using the method called post.

2. The walls of the cavity are moved slightly in and out by means of a adjustable screw which inturn tightens or loosens small bellows. This will result in variation of dimensions of the cavity and then the resonantly frequency

Solved problems on Reflex Klystron

Eg1: The parameter gives for a reflex Klystron are

$V_r = 2kV$, $V_0 = 500V$, $L = 2cm$, $n=1$, $f = 2\ GHz$

Find at the variation in frequency df for dV_r factor = 0.02 or 2%

from equation 5.78

$$df = \frac{0.02 \times 2000}{\dfrac{2\pi \times 2 \times 10^{-2}}{2\pi - \dfrac{\pi}{2}} \sqrt{\dfrac{8 \times 9 \times 10^{-31} \times 500}{1.6 \times 10^{-19}}}}$$

$$df = 10MHz$$

Eg 2. A Reflex klystron operates at the peak mode of n=2

And $V_0 = 300V$

$I_0 = 20mA$

$V_1 = 40V$

Calculate (a) input power in watts 'P_{dc}'

(b) output power in watts 'P_{ac}'

(c) Efficiency n

<u>Solution</u>

(a) Input power in watts
$P_{dc} = V_0 I_0 = 300 \times 20 \times 10^{-3} = 6$ W

(b) Output power in watts, using equation 5.67

$$P_{ac} = \frac{2V_0 I_0 X^1 J_1(\times^1)}{2\pi n - \frac{\pi}{2}}$$

$$Assuming \quad X^1 J_1(\times^1)\int_{max} = 1.25$$

$$P_{ac} = \frac{2 \times 300 \times 20 \times 10^{-3} \times 1.25}{4\pi - \frac{\pi}{2}} = 1.36 walts$$

(c) Efficiency „n"

$$\eta = \frac{P_{ac}}{dc} = \frac{1.36}{6.0} = 0.2267 \; or \; 22.67\%$$

Eg 3. Given parameters of a reflex klystron

V_0 = 400Vm R_{sh} = 20 kΩ,   f = 9GHz

L = 1mm,   n=2

find   (a) Repeller Voltage V_r (b) Efficiency 'η'

Solution : from equation 5.76

$$V_r = V_0 + \frac{2\omega L}{2\pi n - \frac{\pi}{2}} \cdot \sqrt{\frac{2V_0 m}{e}}$$

$$V_r = 400 + \frac{2\pi \times 9 \times 10^9 \times 10^{-3}}{4\pi - \frac{\pi}{2}} \cdot \sqrt{\frac{2 \times 9 \times 10^{-3} \times 400}{1.6 \times 10^{-19}}}$$

$$V_r = 400 + \frac{36\pi \times 10^6 \times 20 \times \sqrt{2 \times 0.569 \times 10^{-11}}}{\frac{7\pi}{2}}.$$

$$V_r = 400 + \frac{720 \times 10^6 \times \sqrt{0.1138 \times 10^{-10}}}{\frac{7}{2}}$$

$$V_r = 1093.9V$$

(c) Efficiency η

From equation - 5.63

$$\eta = \frac{2X^1 J_1(\times^1)}{2\pi n - \frac{\pi}{2}}$$

$$Assuming \; X^1 J(X)\int_{max} = 1.25$$

$$\eta = \frac{2 \times 1.25}{3.5\pi} = 0.227 \; or$$

22.7

UNIT IV

Contents:

- **Cross-field Tubes**
  - Introduction
  - Cross field effects
  - Magnetrons-different types, cylindrical travelling wave magnetron-Hull cutoff and Hartree conditions
- **Microwave Semiconductor Devices:**
  - Introduction to Microwave semiconductor devices, classification, applications
  - Transfer Electronic Devices, Gunn diode - principles, RWH theory, Characteristics, Basic modes of operation - Gunn oscillation modes
  - Introduction to Avalanche Transit time devices (brief treatment only), Illustrative Problems.

Introduction:

Magnetron is a grouping of a simple diode vacuum tube together with built in cavity resonators and an exceptionally powerful magnet. There are three types of magnetrons:

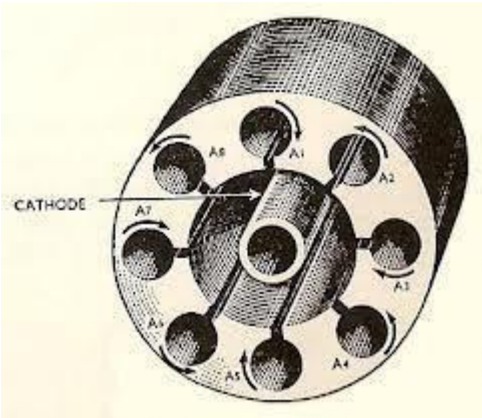
- Negative resistance type
- Cyclotron frequency type
- Travelling wave or Cavity type

Negative resistance magnetrons make use of negative resistance between two anode segments.

Cyclotron frequency magnetrons depends upon synchronism amid an alternating component of electric field and periodic oscillation of electrons in a direction parallel to this field.

Cavity type magnetrons depends upon the interface of electrons with a rotating electromagnetic field of constant angular velocity.

CONSTRUCTION



A magnetron consist of a cathode which is used to release the electrons and number of anode cavities and a permanent magnet is placed on the flipside of cathode and the space between the anode cavity and the cathode is called interacting space.

The electrons which are emitted from the cathode moves in diverse path in the interacting space depending upon strength of electric and magnetic fields applied to the magnetron.

Types of Magnetrons:

There are three main types of Magnetrons.

Negative Resistance Type

- The negative resistance between two anode segments, is used.
- They have low efficiency.
- They are used at low frequencies (< 500 MHz).

Cyclotron Frequency Magnetrons

- The synchronism between the electric component and oscillating electrons is considered.
- Useful for frequencies higher than 100MHz.

Travelling Wave or Cavity Type

- The interaction between electrons and rotating EM field is taken into account.
- High peak power oscillations are provided.
- Useful in radar applications.

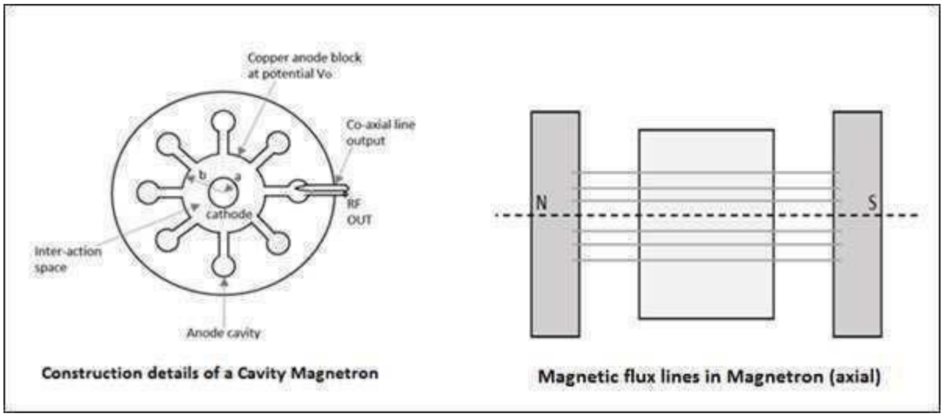
Cavity Magnetron

The Magnetron is called as Cavity Magnetron because the anode is made into resonant cavities and a permanent magnet is used to produce a strong magnetic field, where the action of both of these makes the device work.

Construction of Cavity Magnetron

A thick cylindrical cathode is present at the center and a cylindrical block of copper, is fixed axially, which acts as an anode. This anode block is made of a number of slots that acts as resonant anode cavities.

The space present between the anode and cathode is called as Interaction space. The electric field is present radially while the magnetic field is present axially inthe cavity magnetron. This magnetic field is produced by a permanent magnet, which is placed such that the magnetic lines are parallel to cathode and perpendicular to the electric field present between the anode and the cathode.
The following figures show the constructional details of a cavity magnetron and the magnetic lines of fluxpresent, axially.



Construction details of a Cavity Magnetron

Magnetic flux lines in Magnetron (axial)

This Cavity Magnetron has 8 cavities tightly coupled to each other. An N-cavity magnetron has N modes of operations. These operations depend upon the frequency and the phase of oscillations. The total phase shift around the ring of this cavity resonators should be $2n\pi$ where n is an integer.

If ϕ_V represents the relative phase change of the AC electric field across adjacent cavities, then

$\phi_V = 2\pi n/N$

Where $n = 0, \pm 1, \pm 2, \pm(N/2-1), \pm N/2$

Which means that N/2 mode of resonance can exist if N is

an even number.If,

$n = N/2$ then $\phi_V = \pi$

This mode of resonance is called as $\pi-$mode.

$n = 0$ then $\phi_V = 0$

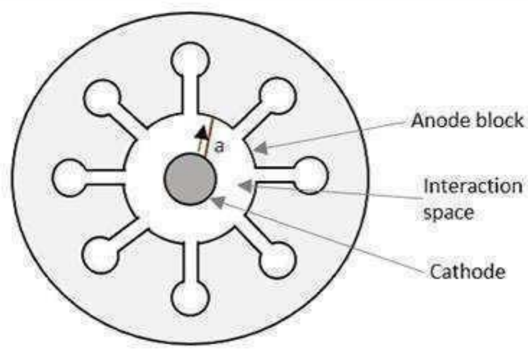
This is called as the Zero mode, because there will be no RF electric field between the anode and the cathode. This is also called as Fringing Field and this mode is not used in magnetrons.

Operation of Cavity Magnetron

When the Cavity Klystron is under operation, we have different cases to consider. Let us go through themin detail.

Case 1

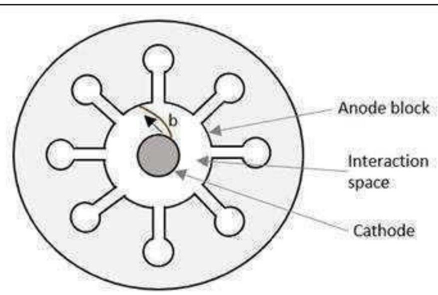
If the magnetic field is absent, i.e. B = 0, then the behavior of electrons can be observed in the following figure. Considering an example, where electron a directly goes to anode under radial electric force.



Movement of electron **a**, when no magnetic field is present

Case 2

If there is an increase in the magnetic field, a lateral force acts on the electrons. This can be observed in the following figure, considering electron b which takes a curved path, while both forces are acting on it.



Movement of electron **b** when small magnetic field is present

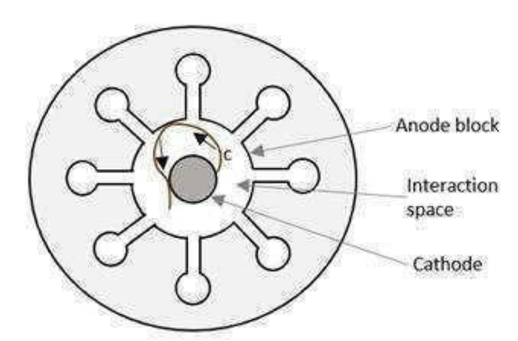
Radius of this path is

calculated as$R=mv/eB$

It varies proportionally with the velocity of the electron and it is inversely proportional to the magneticfield strength.

Case 3

If the magnetic field B is further increased, the electron follows a path such as the electron c, just grazing the anode surface and making the anode current zero. This is called as "Critical magnetic field" (B_c), which is the cut-off magnetic field. Refer the following figure for better understanding.

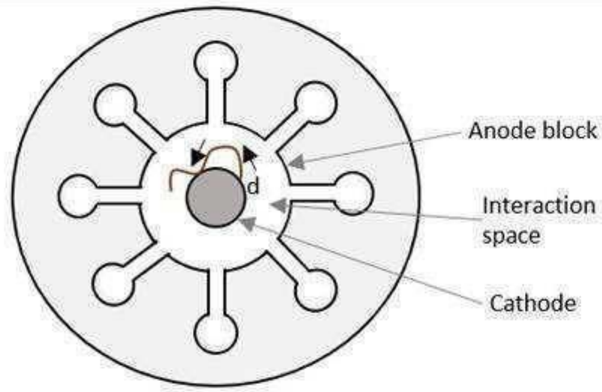


Movement of electron **c** when the magnetic field is critical magnetic field i.e. $B = B_c$

Case 4

If the magnetic field is made greater than the critical field,$B>B_c$

Then the electrons follow a path as electron d, where the electron jumps back to the cathode, without going to the anode. This causes "back heating" of the cathode. Refer the following figure.



Movement of electron **d** when excessive magnetic field is present

This is achieved by cutting off the electric supply once the oscillation begins. If this is continued, theemitting efficiency of the cathode gets affected.

Operation of Cavity Magnetron with Active RF Field

We have discussed so far the operation of cavity magnetron where the RF field is absent in the cavities ofthe magnetron (static case). Let us now discuss its operation when we have an active RF field.

As in TWT, let us assume that initial RF oscillations are present, due to some noise transient. The oscillations are sustained by the operation of the device. There are three kinds of electrons emitted in this process, whose actions are understood as electrons a, b and c, in three different cases.

Case 1

When oscillations are present, an electron a, slows down transferring energy to oscillate. Such electrons that transfer their energy to the oscillations are called as favored electrons. These electrons are responsiblefor bunching effect.

Case 2

In this case, another electron, say b, takes energy from the oscillations and increases its velocity. As and when this is done,

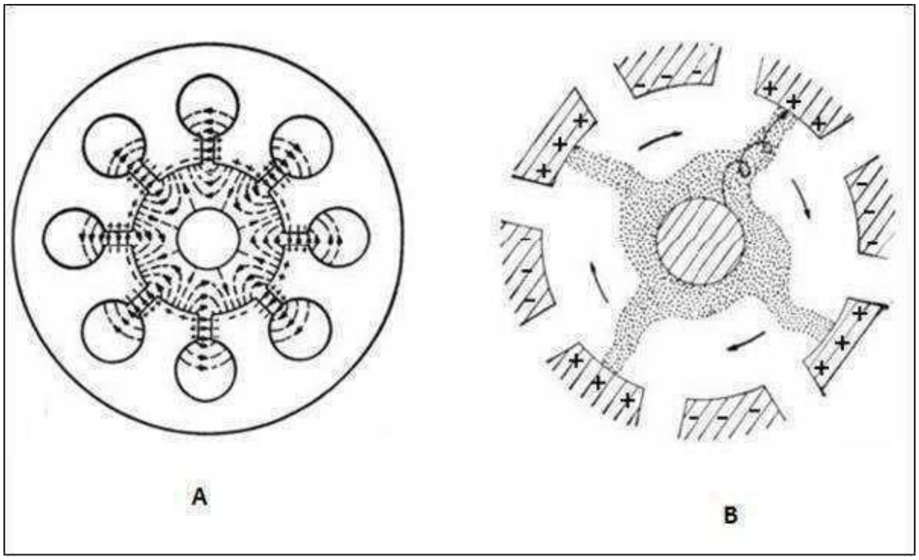
- It bends more sharply.
- It spends little time in interaction space.
- It returns to the cathode.

These electrons are called as unfavored electrons. They don't participate in the bunching effect. Also, these electrons are harmful as they cause "back heating".

Case 3

In this case, electron c, which is emitted a little later, moves faster. It tries to catch up with electron a. The next emitted electron d, tries to step with a. As a result, the favored electrons a, c and d form electron bunches or electron clouds. It called as "Phase focusing effect".

This whole process is understood better by taking a look at the following figure.



Phase Focusing Effect

Figure A shows the electron movements in different cases while figure B shows the electron clouds formed.
These electron clouds occur while the device is in operation. The charges present on the internal surface of these anode segments, follow the oscillations in the cavities. This creates an electric field rotating clockwise, which can be actually seen while performing a practical experiment.

While the electric field is rotating, the magnetic flux lines are formed in parallel to the cathode, under whose combined effect, the electron bunches are formed with four spokes, directed in regular intervals, to the nearest positive anode segment, in spiral trajectories.

INTRODUCTION

The application of two-terminal semiconductor devices at microwave frequencies has been increased usage during the past decades. The CW, average, and peak power outputs of these devices at higher microwave frequencies are much larger than those obtainable with the best power transistor.

The common characteristic of all active two-terminal solid-state devices is their negative resistance. The real part of their impedance is negative over a range of frequencies.

In a positive resistance the current through the resistance and the voltage across it are in phase. The voltage drop across a positive resistance is positive and a power of (I2 R) is dissipated in the resistance.

In a negative resistance, however, the current and voltage are out of phase by 180°. The voltage drop across a negative resistance is negative, and a power of ($-I^2R$) is generated bythe power supply associated with the negative resistance.

In other words, positive resistances absorb power (passive devices), whereas negative resistances generate power (active devices).

The differences between microwave transistors and transferred electron devices (TEDs) are fundamental. Transistors operate with either junctions or gates, but TEDs are bulk devices having no junctions or gates. The majority of transistors are fabricated from elemental semiconductors, such as silicon or germanium, whereas TEDs are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or cadmium telluride (CdTe).

TEDS operate with ‘hot’ electrons whose energy is much greater than thermal energy.

GUNN DIODES-GaAs DIODE

Gunn Diode is a one kind of transferred electronic device and exhibits negative resistance characteristic.

Gunn-effect diodes are named after J. B. Gunn, who in 1963 discovered periodic fluctuations of current passing through then-type gallium arsenide (GaAs) specimen when the applied voltage exceeded a certain critical value.

These are bulk devices in the sense that microwave amplification and oscillation are derived from the bulk negative-resistance property of uniform semiconductors rather than from the junction negative-resistance property between two different semiconductors, as in the tunnel diode.

GUNN EFFECT:

A schematic diagram of a uniform n-type GaAs diode with ohmic contacts at the end surfacesare shown in Fig.1.

J. B. Gunn observed the Gunn effect in the n-type GaAs bulk diode in 1963.

Above some critical voltage, corresponding to an electric field of 2000-4000 volts/cm, the current in every specimen became a fluctuating function of time. In the GaAs specimens, this fluctuation took the form of a periodic oscillation superimposed upon the pulse current. The frequency of oscillation was determined mainly by the specimen, and not by the external circuit. The period of oscillation was usually inversely proportional to the specimen length and closely equal to the transit time of electrons between the electrodes, calculated from their estimated velocity of slightly over 107 cm/s.

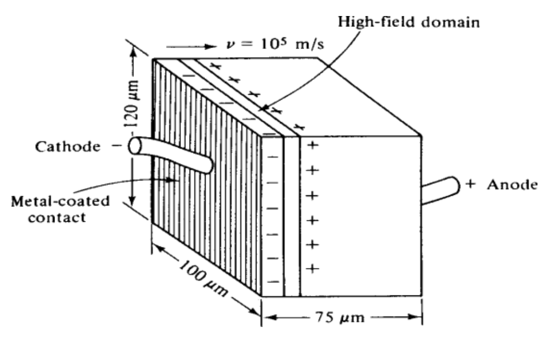


Figure 1. Schematic diagram for ntype GaAs diode.

From Gunn's observation the carrier drift velocity is linearly increased from zero to a maximum when the electric field is varied from zero to a threshold value. When the electric field is beyond the threshold value of 3000 V/cm for the n-type GaAs, the drift velocity is decreased and the diode exhibits negative resistance.

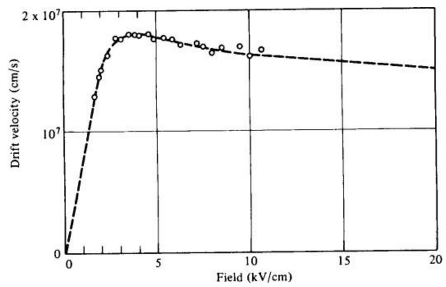


Figure 2. Drift velocity of electrons in n-type GaAs versus electric field.

RIDLEY-WATKINS-HILSUM (RWH) THEORY

The fundamental concept of the Ridley-Watkins-Hilsum (RWH) theory is the differential negative resistance developed in a bulk solid-state III-V compound when either a voltage (or electric field) or a current is applied to the terminals of the sample. There are two modes of negative-resistance devices: voltage-controlled and current-controlled modes as shown in Fig.3.a and Fig.3.b

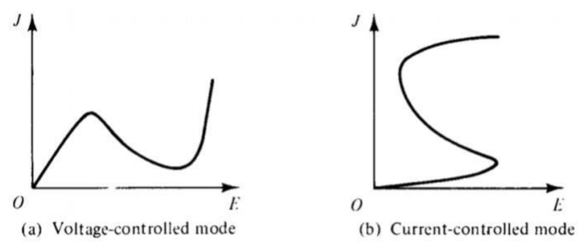


Figure 3. Diagram of negative resistance

In the voltage-controlled mode the current density can be multivalued, whereas in the current- controlled mode the voltage can be multivalued. The major effect of the appearance of a differential negative-resistance region in the current density-field curve is to render the sample electrically unstable. As a result, the initially homogeneous sample becomes electrically heterogeneous in an attempt to reach stability. In the voltage-controlled negative- resistance mode high-field domains are formed, separating two low-field regions. The interfaces separating low and high- field domains lie along equipotential; thus they are in planes perpendicular to the current direction as shown in Fig. 4(a). In the current-controlled negative-resistance mode splitting the sample results in high-current filaments running along the field direction as shown in Fig. 4(b).

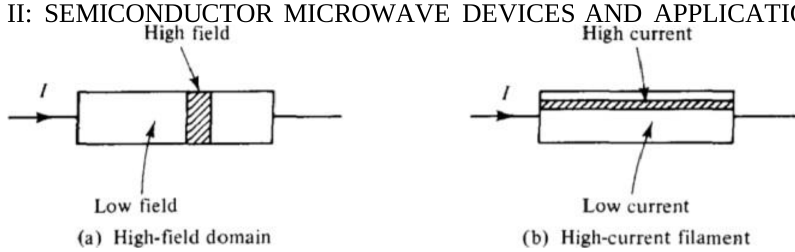


Figure 4.Diagrams of high field domain and high current filament.

Expressed mathematically, the negative resistance of the sample at a particular region is

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance} \qquad ..(1)$$

If an electric field E0 (or voltage V0) is applied to the sample, for example, the current density lo is generated. As the applied field (or voltage) is increased to E2(or V2), the current density is decreased to J2. When the field (or voltage) is decreased to E1 (or V1), the current density is increased to J1 . These phenomena of the voltage-controlled negative resistance are shown in Fig. 5(a). Similarly, for the current-controlled mode, the negative- resistance profile is as shown in Fig. 5(b).

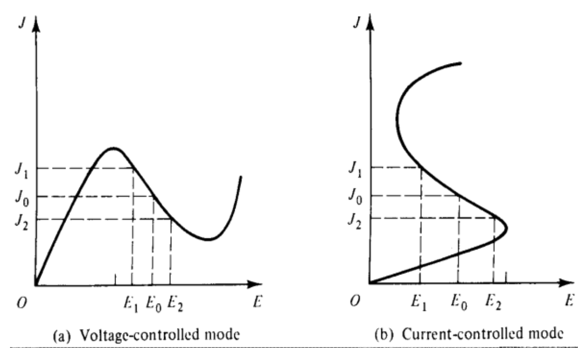


Figure 5.Multiple values of current density for negative resistance.

TWO-VALLEY MODEL THEORY

According to the energy band theory of then-type GaAs, a high-mobility lower valley is separated by an energy of 0.36 eV from a low-mobility upper valley as shown in Fig. 6.

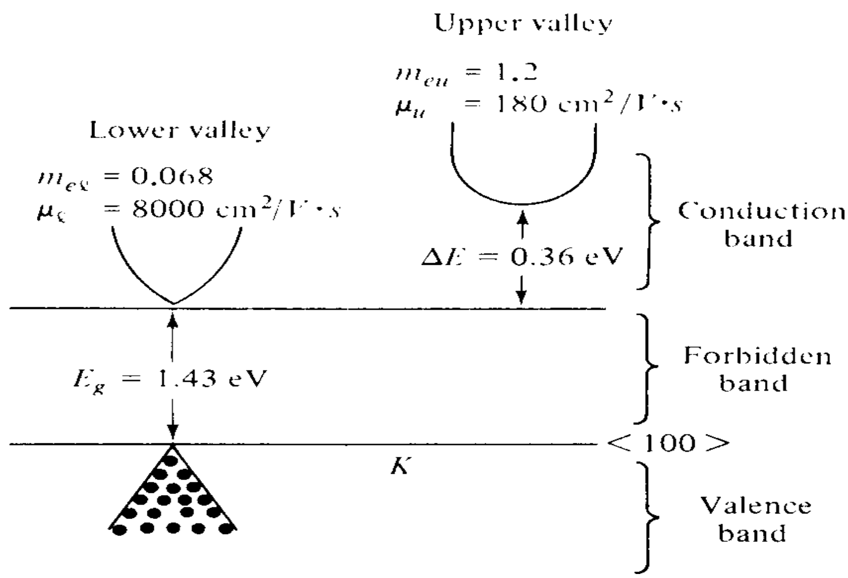


Figure 6.Two-valley model of electron energy versus wave number for n-type GaAs.

TABLE 1. DATA FOR TWO VALLEYS IN GaAs

Valley	Effective MassMe	Mobility μ	Separation ΔE
Lower	Mel = 0.068	μl = 8000 cm2/v-sec	ΔE = 0.36 eV
Upper	Meu= 1.2	μu = 180 cm2/v-sec	ΔE = 0.36 eV

Electron densities in the lower and upper valleys remain the same under an equilibrium condition. When the applied electric field is lower than the electric field of the lower valley (E<Ee), no electrons will transfer to the upper valley as shown in Fig. 7(a).

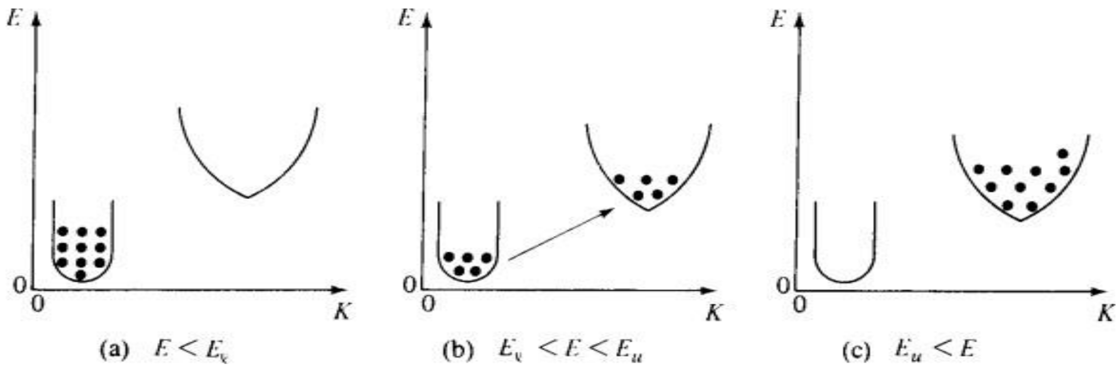


Figure 7 Transfer of electron densities.

When the applied electric field is higher than that of the lower valley and lower than that of the upper valley (Ee<E<Eu), electrons will begin to transfer to the upper valley as shown in Fig. 7(b). And when the applied electric field is higher than thatof the upper valley (Eu<E), all electrons will transfer to the upper valley as shown in Fig. 7(c). If electron densities in the lower and upper valleys are ne and nu , the conductivity of the n -type GaAs is

$\sigma = e(n_l \mu_l + n_u\mu_u)$ ..(2) Where,e = the electron chargeμ = electron mobilityn

= ne + nu is the electron density

When a sufficiently high field Eis applied to the specimen, electrons are accelerated and their effective temperature rises above the lattice temperature. Furthermore, the lattice temperature also increases. Thus electron density n and mobility μ are both functions of electric field E.

Differentiation of Eq. (7-2-2) with respect to E yields

$$\frac{d\sigma}{dE} = e\left(\mu_l \frac{dn_l}{dE} + \mu_u \frac{dn_u}{dE}\right) + e\left(n_l \frac{d\mu_l}{dE} + n_u \frac{d\mu_u}{dE}\right) \qquad ..(3)$$

If the total electron density is given by $n = n_l + n_u$and it is assumed that $\mu_l$andμ_u areproportional to Ep, where p is a constant, then

$$\frac{d}{dE}(n_l + n_u) = \frac{dn}{dE} = 0 \quad ..(4)$$

$$\frac{dn_l}{dE} = -\frac{dn_u}{dE} \quad ..(5)$$

and $$\frac{d\mu}{dE} \propto \frac{dE^p}{dE} = pE^{p-1} = p\frac{E^p}{E} \propto p\frac{\mu}{E} = \mu\frac{p}{E} \quad ..(6)$$

Substitution of Equation (4) to (6) into Eq. (3) results in

$$\frac{d\sigma}{dE} = e(\mu_l - \mu_u)\frac{dn_l}{dE} + e(n_l\,\mu_l + n_u\mu_u)\frac{p}{E} \ldots(7)$$

Then differentiation of Ohm's law $J = \sigma E$ with respect to E yields

$$\frac{dJ}{dE} = \sigma + \frac{d\sigma}{dE}E \quad ..(8)$$

Equation (8) can be rewritten

$$\frac{1}{\sigma}\frac{dJ}{dE} = 1 + \frac{d\sigma/E}{\sigma/E} \quad ..(9)$$

Clearly, for negative resistance, the current density J must decrease with increasing field E or the ratio of dJ/dE must be negative. Such would be the case only if the right-hand term of Eq.
(9) is less than zero. In other words, the condition for negative resistance is

$$-\frac{d\sigma/E}{\sigma/E} > 1 \quad ..(10)$$

Substitution of Equation (2) and (7) with f = nu/nl results in [2]

$$\left[\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\left(-\frac{E}{n_l}\right)\frac{dn_l}{dE} - p\right] > 1 \quad ..(11)$$

APPL6I·CATION OF GUNN DIODE

- In Radar Transmitters (police Radar, CW Doppler radar).
- Pulsed Gunn diode oscillators used in transponders, for air traffic control and in industrytelemetry system.
- Fast combinational and sequential logic amplifier. As pump source in preamplifier.
- In microwave receiver as low and medium power oscillator.

Domain Formation:

Differential resistance occur when an electric field of a certain range is applied to a multivalley semiconductor, such as then-type GaAs. due to that decrease in drift velocity with increasing electric field. Its leads to formation of a high-field domain for microwave generation andamplification.

In the n-type GaAs diode the majority carriers are electrons. When a small voltage is applied to the diode, the electric field and conduction current density are uniform diode.

$$J = \sigma E_x = \frac{\sigma V}{L} U_x = \rho \vartheta_x U_x$$

Where

J = conduction current density σ = conduc~~ti~~vi$_x$ty

Ex= electric field in the x direction

L = length of the diode

V = applied voltage

p = charge density

v = drift velocity

U = unit vector

The current is carried by free electrons that are drifting through a background of fixed positive charge.

When the applied voltage is above the threshold value, which measuredabout 3000 V/cmtimes the thickness of the GaAs diode, a high-field domain is formed near the cathode that reduces the electric field.

$$V = -\int_0^L E_x \, dx$$

The high field domain then drifts with the carrier stream across the electrodes and disappears at the anode contact. When the electric field increases, the electron drift velocity decreases and the GaAs exhibits negative resistance.

As shown fig 1(b) below there exists an excess (or accumulation) of negative charge that could be caused by a random noise fluctuation or possibly by a permanent nonuniformity in doping in the n-type GaAs diode.

An electric field is then created by the accumulated charges as shown in Fig 1(d). The fieldto the left of point A is lower than that to the right. If the diode is biased at point EA on the J-E curve, implies that the carriers (or current) flowing into point A are greater than those flowing out of point A,therefore increasing the excess negative space charge at A.

when the electric field to the left of point A is lower than it was before, the field to the right is then greater than the original one, resulting in an even greater space-charge accumulation. process continues until the low and high fields both reach outside the differential negative- resistance region Fig1(a).

This process depends on condition that the number of electrons inside the crystal is large enough to allow the necessary amount of space charge to be built up during the transit time of the space-charge layer.

The electric field in side the dipole domain would be greater than the fields on either side of the dipole in Fig 2.(c). Because of the negative differential resistance, the current in the low- field side would be greater than that in the high-field side.

Then the dipole field reaches a stable condition and moves through the specimen toward the anode. When the high-field domain disappears at the anode, a new dipole field starts formingat the cathode and the process is repeated.

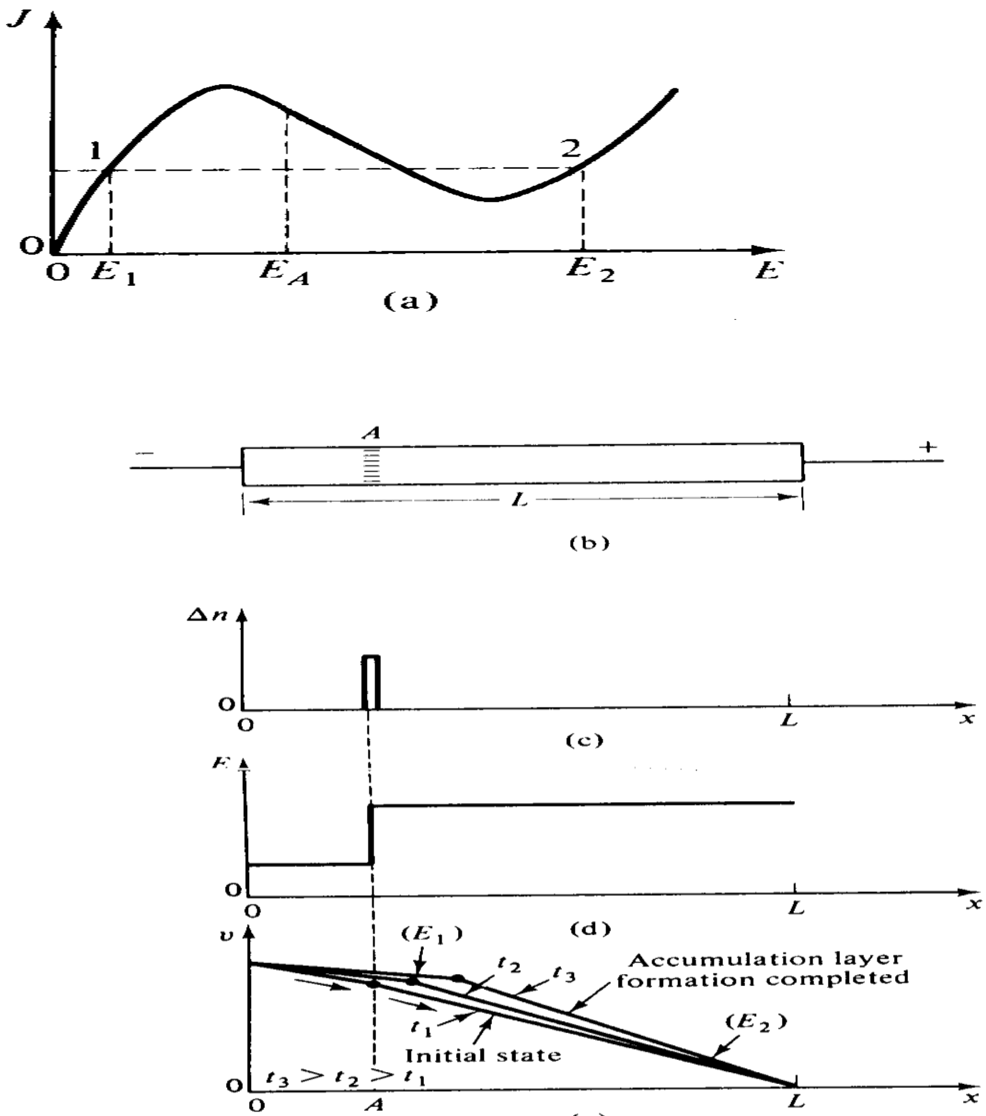


Figure 1: formation of an electron accumulation layer in GaAs.

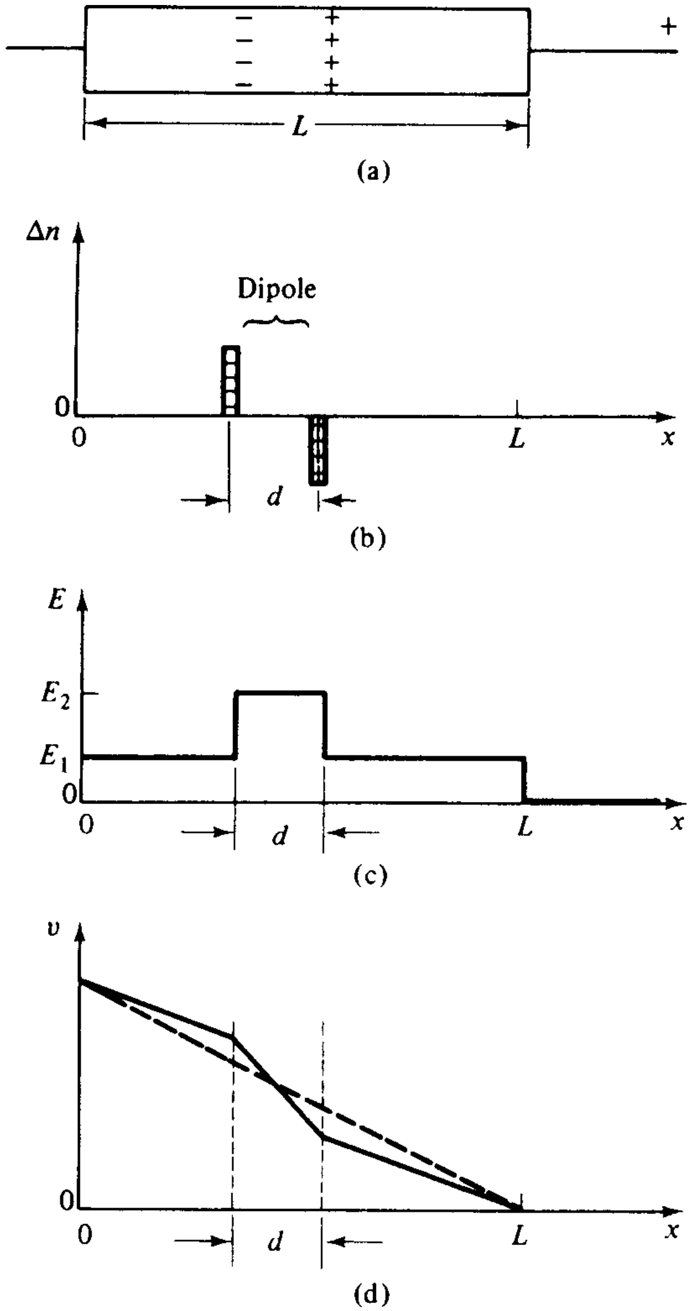


Figure2: formation of an electron dipole layer in GaAs.

Properties of High field domain

Will start to form whenever the electric field in a region of the sample increases above the threshold E.When the electric field increases, the electron drift velocity decreases and the GaAs diode exhibits negative resistance.

If additional voltage is applied, the domain will increase in size and absorb more voltage than was added and the current will decrease.

domain will not disappear before reaching the anode unless the voltage is dropped appreciably below threshold.

New domain formation can be prevented by decreasing the voltage slightly below threshold.

Domain will modulate the current through a device as the domain passes through regions of different doping and cross-sectional area,or domain may disappear. Effective doping may varied in region.

The domain length is inversely proportional to the doping. devices with the same product of dopingmultiplied by length will behave similarly in terms of frequency multiplied by length.

Domain can be detected by a capacitive contact.Presence of a domain anywhere in a devicecan be detected by a decreased current.

Modes of operation of Gunn diode

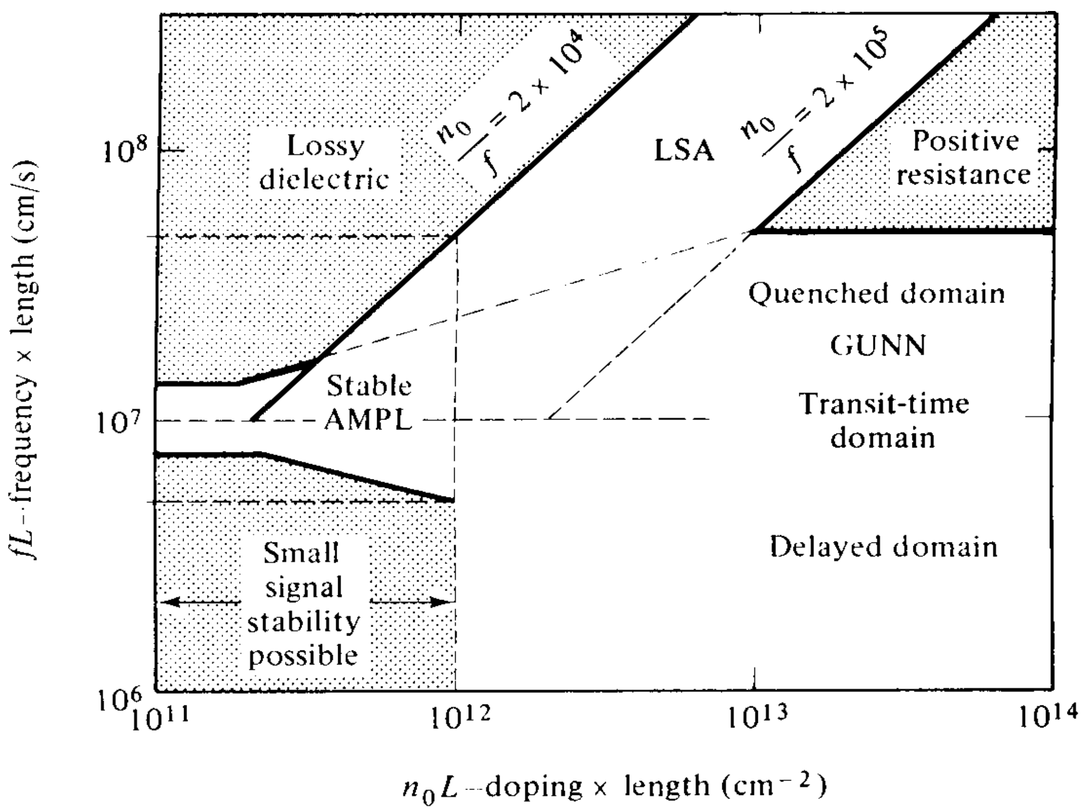


Figure 3: Modes of operation for Gunn

Gunn first announced his observation of microwave oscillation in the n-type GaAs and n-type InP diodes in 1963, various modes of operation have been developed, depending on the material parameters and operating conditions.

Formation of strong space-charge instability depends on the conditions that enough charge isavailable in the crystal.

four basic modes of operation of uniformly doped bulk diodes with low-resistance contactare as follows

- Transit Time Domain mode
- Delayed Domain mode
- Quenched Domain mode
- Limited space charge accumulation mode

1. Transit Time Domain mode:

in the region where the product of frequency multiplied by length is about 107 cm/s and the product of doping multiplied by length is greater than l012/cm2.the device is unstable because of the cyclic formation of either the accumulation layer or the high-field domain.

f = Vd/L in these mode is slightly sensitive to the applied voltages since the drift velocity Vddepends on the bias voltages.

Vd= f .L = 107 cm/s when Vd = VS, thenhigh field domain is stable. Bias voltage is normally maintain little higher Eth .

At this instant Oscillation period = Transit Time (τo = τt). Operating 'f' depends on 'Vd' hence on bias voltage > Eth.

It is a low power,low efficiency mode and requires that operating frequency laser then30GHz.

These limit on frequency is due to that device length .

2. Delayed Domain mode:

This mode is defined in the region where the product of frequency times length is about 107 cm/s and the product of doping times length is between 1011 and l012/cm2.

When transit time is chosen that domain is collected E <Eth ,new domain can not form untilfield rises again above threshold.

Oscillation period is greater than transit time $\tau o < \tau t$This dealy inhabited mode has an 20 %

Efficiency.

Operating frequency can be less than or equal to Gunn Mode frequency

3. Quenched Domain mode:

This mode is defined in the region where the product of frequency times length is above 107 cm/s and the quotient of doping divided by frequency is between 2 x 104and 2 x 105.

It is bias field drops below sustaining field Esduring the negative half cycle domain collapses before it reaches the anode.i.e The domain disappear somewhere in the sample itself.

Operating frequency will be higher than Gunn Mode and delayed mode, certainly this dependon the external circuit.

When bias field swings back above threshold value Vth, new domain formed and process repeats,hence in that mode domain is quenched before it reches the anode.

Frequency of resonant circuit then the transit time frequency is 13%.

4. Limited Space Charge Accumulation mode:

This mode occurs only when there is either Gunn or LSA oscillation, and it is usually at the region where the product of frequency time's length is too small to appear in the figure. When a bulk diode is biased to threshold, the average current suddenly drops as Gunn oscillation begins.

It gives high power upled high efficiencythe domain is not allowed to form

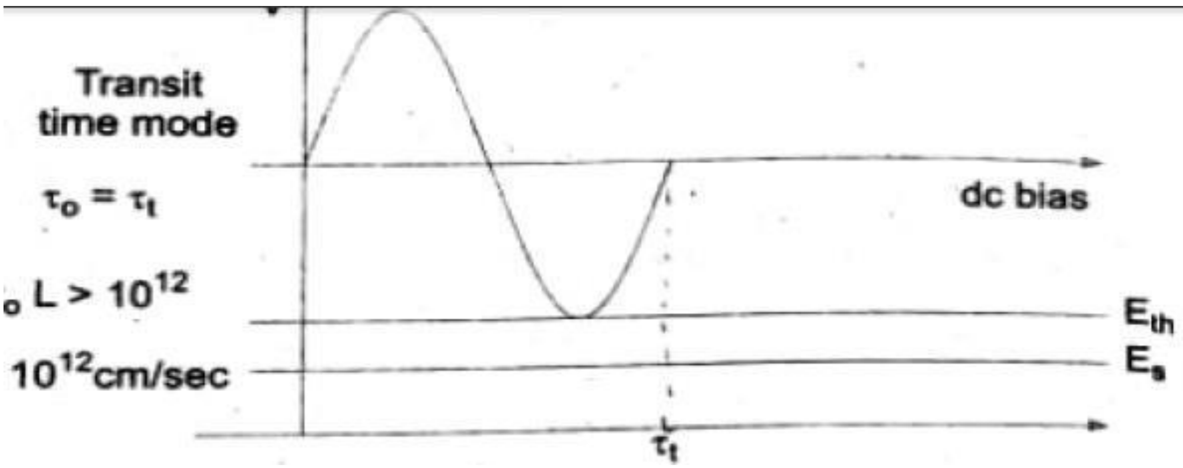
RF voltage and frequency are so chosen that they do not have sufficient time to form domainabove threshold.

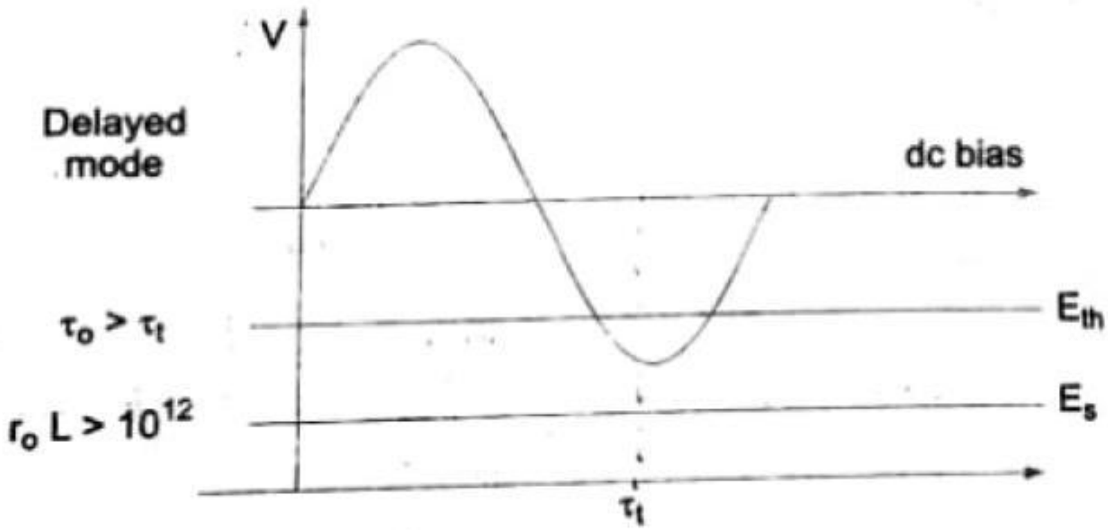
IN LSA mode high power and high $\eta(20\%)$,16 to 23% compare to 5% for gunn mode Thefield No peak value permits high operating voltage.

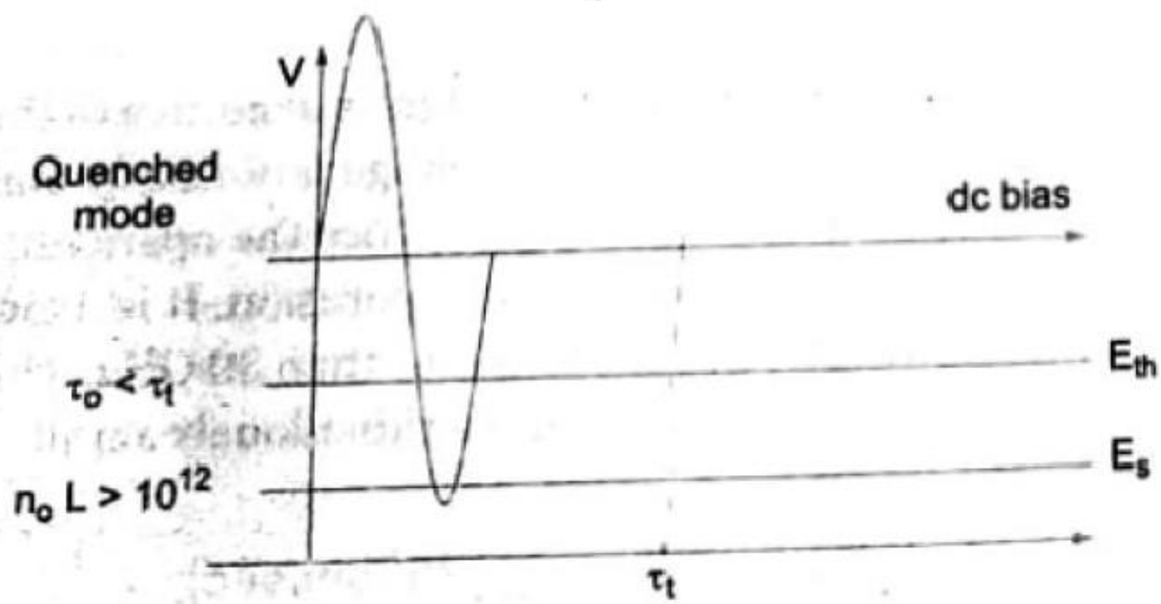
Operating frequency is 0.5-50 times more than Gunn Mode.

It can be used up to 100 GHz and is dependent on external resonating circuits.High

Power and High Efficiency.


Transit time mode
$\tau_o = \tau_t$
$_o L > 10^{12}$
10^{12}cm/sec
dc bias
E_{th}
E_s
τ_t


V
Delayed mode
$\tau_o > \tau_t$
$r_o L > 10^{12}$
dc bias
E_{th}
E_s
τ_t


V
Quenched mode
$\tau_o < \tau_t$
$n_o L > 10^{12}$
dc bias
E_{th}
E_s
τ_t

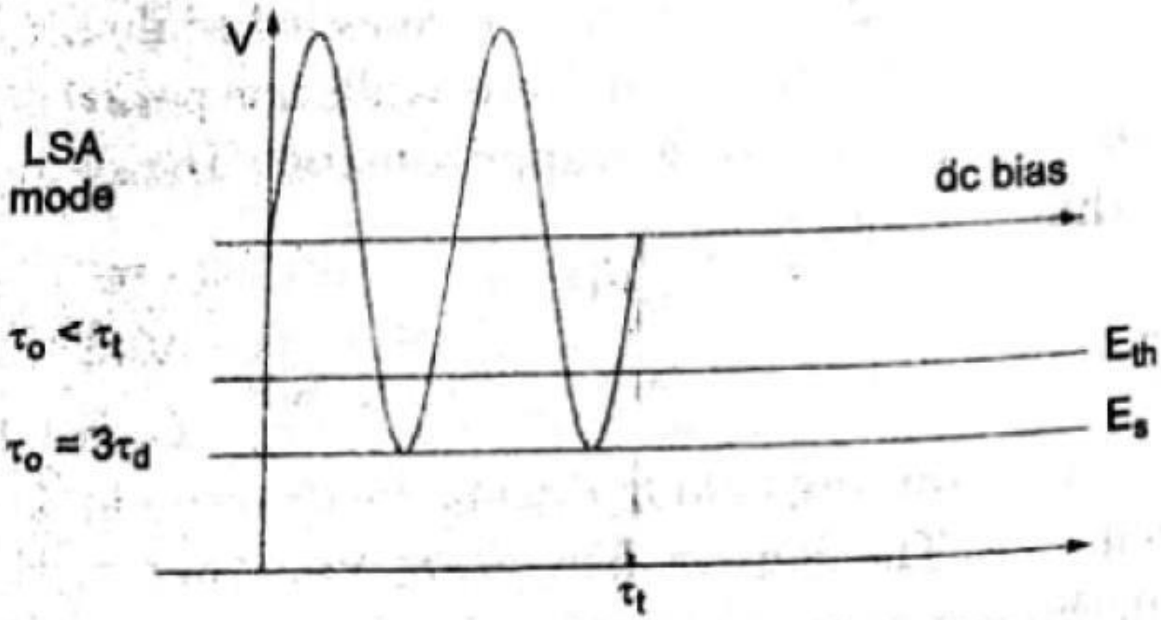


Fig. 9.47 Gunn diode modes.

UNIT-V

MICROWAVE MEASUREMENTS

Contents:

- Description of Microwave Bench – Different Blocks and their Features,
- Waveguide Attenuators – Resistive Card, Rotary Vane types;
- Microwave Power Measurement – Bolometer Method.
- Measurement of Attenuation, Frequency, VSWR
- Impedance Measurements.

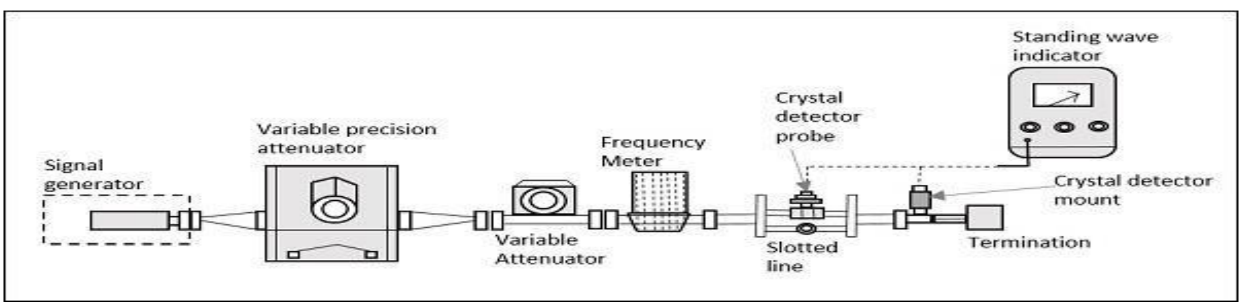
Introduction:

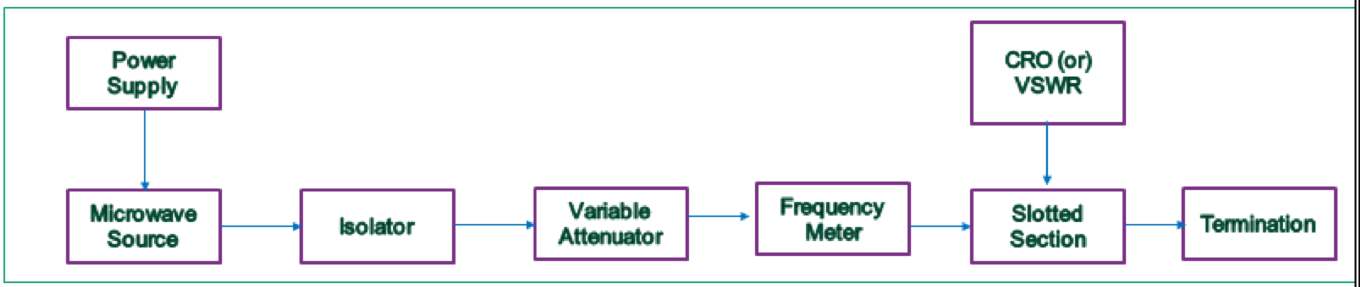
- Measurement of voltage and current is easy at Low frequencies therefore power calculation is easy where as at microwave frequencies it is difficult to measure voltage and current as they are vary with position in a transmission line.
- So at microwave frequencies it is convenient to measure power directly instead of voltage and current.
- At low frequencies, circuits use lumped elements which can be identified and measured. At microwave frequencies, circuit elements are distributed and hence it is not important to know what element make up a line.
- Unlike low frequency measurements, many quantities measured at microwave frequencies are relative and it is not necessary to know their absolute values.
- Further for power measurement, it is usually sufficient to know the ratio of two powers rather than exact input or output powers

The following parameters can be conveniently measured at microwave frequencies

- Frequency
- Power
- Attenuation
- Voltage Standing Wave Ratio (VSWR)
- Phase
- Impedance
- Insertion Loss
- Dielectric Constant
- Noise factor

Microwave Bench Block diagram:





The general set up for measurement of any parameter in microwaves is normally don by a microwave bench.

Power Supply:

- The power supply gives necessary beam voltage and beam current to the circuit. Also repeller voltage delivered by this unit.
- In lab typically we use 300V beam voltage, 24 mA beam current and take output readings by varying repeller voltage from -50V to 270V.

Microwave Source:

- The source of microwave may be Gunn diode oscillator, Reflex Klystron or BWO.
- Microwave source can provide either a continuous wave (CW) or square wave modulated at an audio rate which is normally 1KHz.

Isolator:

- Isolator is used to protect the source from the reflected power due to mismatch of the load.
- Power flows in only one direction from source to load.

Precision Attenuator or Variable Attenuator:

- The precision attenuator can provide 0 to 50 dB attenuation above insertion loss.
- The variable flat attenuator is also used in addition, whose calibration can be checked against readings of the precision attenuator.

Frequency Meter:

This is the device which measures the frequency of the signal. With this frequency meter, the signal can be adjusted to its resonance frequency. It also gives provision to couple the signal to waveguide.

Crystal Detector:

A crystal detector probe and crystal detector mount are indicated in the above figure, where the detector is connected through a probe to the mount. This is used to demodulate the signals.

Slotted Line:

In a microwave transmission line or waveguide, the electromagnetic field is considered as the sum of incident wave from the generator and the reflected wave to the generator. The reflections indicate a mismatch or a discontinuity. The magnitude and phase of the reflected wave depends upon the amplitude and phase of the reflecting impedance.

The standing waves obtained are measured to know the transmission line imperfections which are necessary to have knowledge on impedance mismatch for effective transmission. This slotted line helps in measuring the standing wave ratio of a microwave device.

Standing Wave Integrator:

It is an element that reinforces producing the reading classification of the standing wave proportion. It gives and slots the waveguide with the help of a gap to adjust the clock cycles of the given signal. It forwards the movements through BNC cable to CRO or VSWR to estimate the general characteristics.

Frequency Meter:

It is a component that measures the frequency of the given signal, and it adjusts to its resonance frequency. The frequency meter also delivers regulation from the motion to the waveguide.

Waveguide Attenuators:

- Attenuator is an electronic device that reduces the power of the signalwithout effecting or reducing the waveform of the signal.
- A device used to control the amount of microwave power transferred from one point to another on a microwave transmission systems is called microwave attenuator.
- Microwave attenuators control the flow of microwave power either by reflecting it or absorbing it.
- Attenuators are commonly used for
    - Measuring power gain or loss in dB
    - Providing isolation between instruments
    - Reducing the power I/P to a particular stage to prevent overloading.

Attenuators can be classified as fixed or variable type

1. **Fixed Attenuators:**

- Fixed attenuators in circuits are used to lower voltage, dissipate power and to impedance matching.

- These are used where fixed amount of attenuation is to provided. If such a fixed attenuator absorbs all the energy entering into it, we call it as a waveguide terminator.
- This normally consists of a short section of waveguide with a tapered plug of absorbing material at the end.
- The tapering is done for providing a gradual transition from the wave guide medium to the absorbing medium thus reducing the reflection occurring at the media interface.

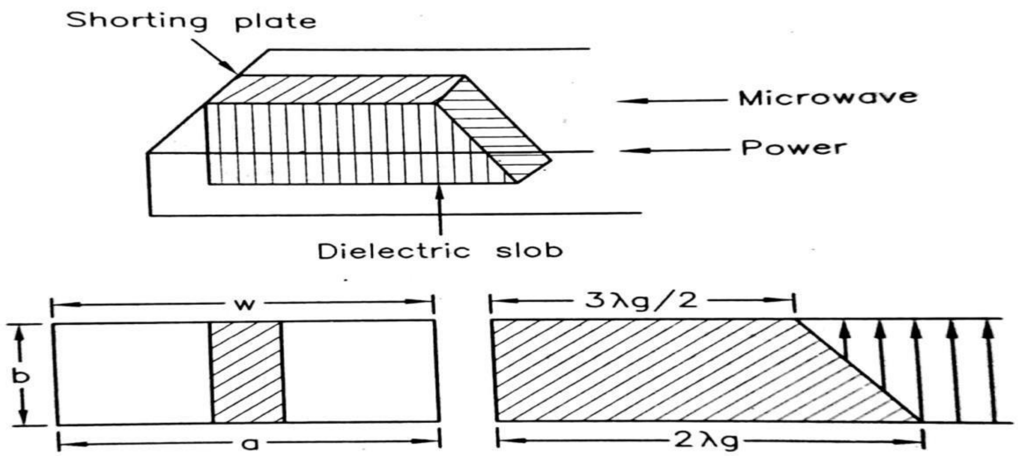


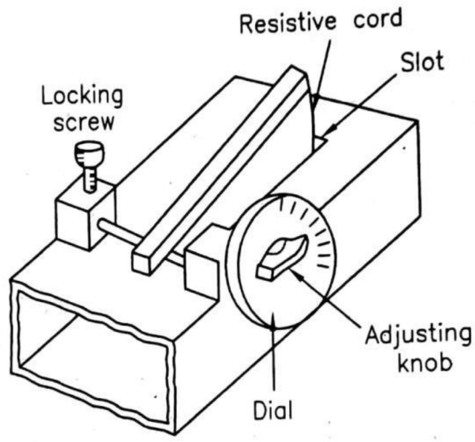
Figure shows fixed attenuator where a dielectric slab consisting of glass slab coated with aquadag or carbon film has been used as a plug.

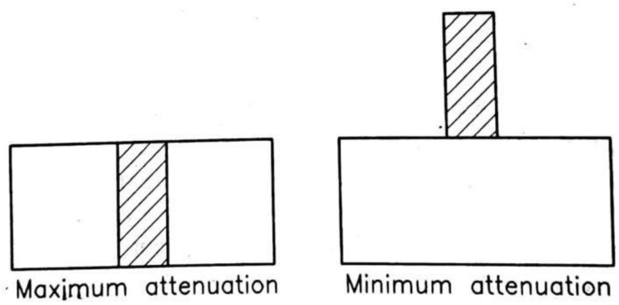
2. **Variable Attenuators:**

- Variable attenuators provide continuous or step wise variable attenuation.
- For rectangular waveguides, these attenuators can be flap type or vane type.
- For circular waveguide rotary type is used.

Resistive or Flap type Attenuators:

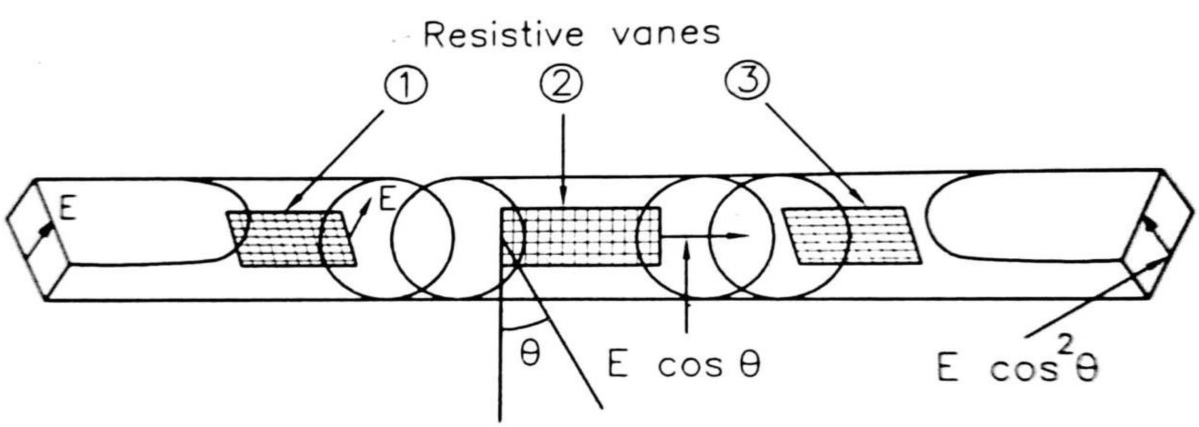
- Flap type attenuator consists of a resistive element or disc inserted into a longitudinal slot cut along the center of the wider dimension of the guide.
- Flap is mounted on the hinged arm allowing it to descent into the centre of waveguide.
- Degree of attenuation can be determined by depth of insertion of the flap.





Rotatory vane Attenuator:

- A resistive rotary vane attenuator consists of three vanes.
- The central vane rotating type placed in the central section of a circular waveguide arrangement tapered at both ends.
- The other two vanes are rectangular sections.

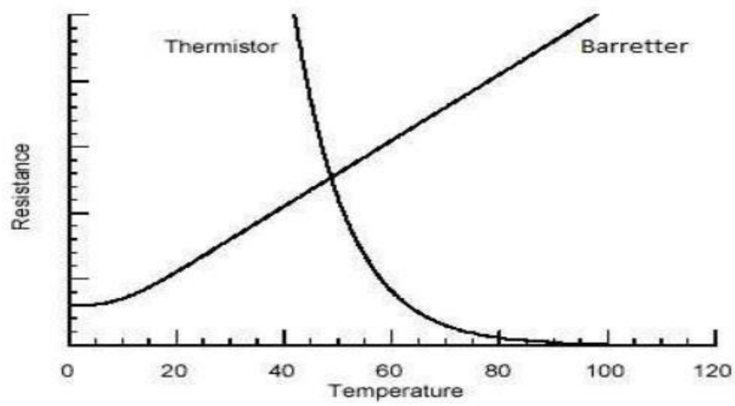


Microwave Power Measurement:

- The Microwave Power measured is the average power at any position in waveguide.
- The measurement of power can be divided into three categories
  - Measurement of Low microwave power (o.o1 mW-10 mW) – Bolometer technique
  - Measurement of medium microwave power (10 mW – 1 W) – Calorimetric Technique
  - Measurement of high power microwave (> 10 W) – Calorimetric Watt meter

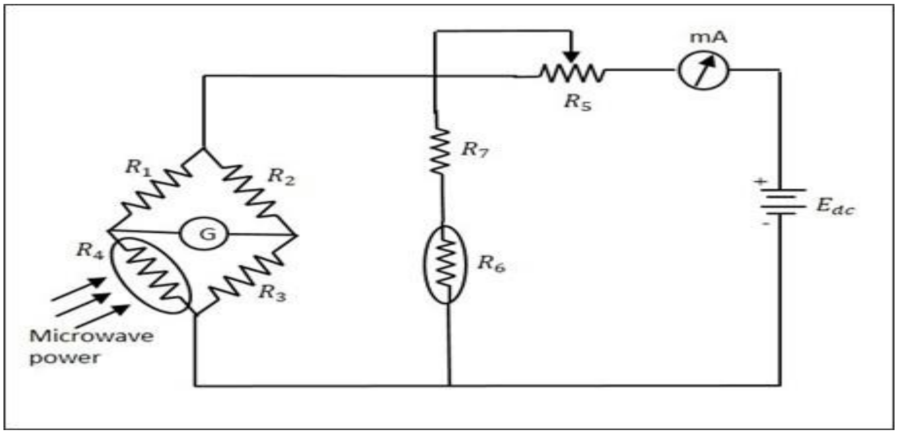
Measurement of Low microwave power:

- Devices such as bolometers and thermocouples whose resistance changes with the applied power are capable of measuring low microwave powers.
- Bolometer is a simple temperature sensitive device whose resistance varies with temperature.
- Bolometers are two types i.e., Barretters and Thermisters.
- Barretters have positive temperature coefficient and their resistance increases with an increase in temperature



Bolometer is a device which is used for low Microwave power measurements. The element used in bolometer could be of positive or negative temperature coefficient. For example, a barrater has a positive temperature coefficient whose resistance increases with the increase in temperature. Thermistor has negative temperature coefficient whose resistance decreases with the increase in temperature.

Any of them can be used in the bolometer, but the change in resistance is proportional to Microwave power applied for measurement. This bolometer is used in a bridge of the arms as one so that any imbalance caused, affects the output. A typical example of a bridge circuit using a bolometer is as shown in the following figure.



The millimeter here, gives the value of the current flowing. The battery is variable, which is varied to obtain balance, when an imbalance is caused by the behavior of the bolometer. This adjustment which is made in DC battery voltage is proportional to the Microwave power. The power handling capacity of this circuitis limited.

Measurement of Medium Power

The measurement of Microwave power around 10mW to 1W, can be understood as the measurement of medium power.

A special load is employed, which usually maintains a certain value of specific heat. The power to be measured, is applied at its input which proportionally changes the output temperature of the load that it already maintains. The difference in temperature rise, specifies the input Microwave power to the load.

The bridge balance technique is used here to get the output. The heat transfer method is used for the measurement of power, which is a Calorimetric technique.

Measurement of High Power

The measurement of Microwave power around 10W to 50KW, can be understood as the measurement of high power.

The High Microwave power is normally measured by Calorimetric watt meters, which can be of dry and flow type. The dry type is named so as it uses a coaxial cable which is filled with di-electric of high hysteresis loss, whereas the flow type is named so as it uses water or oil or some liquid which is a good absorber of microwaves.

The change in temperature of the liquid before and after entering the load, is taken for the calibration of values. The limitations in this method are like flow determination, calibration and thermal inertia, etc.

Measurement of Attenuation

In practice, Microwave components and devices often provide some attenuation. The amount of attenuation offered can be measured in two ways. They are − Power ratio method and RF substitution method.

Attenuation is the ratio of input power to the output power and is normally expressed in decibels.

$$Attenuation\ in\ dBs = 10\ log \frac{P_{in}}{P_{out}}$$

Where P_{in} = Input power and P_{out} = Output power

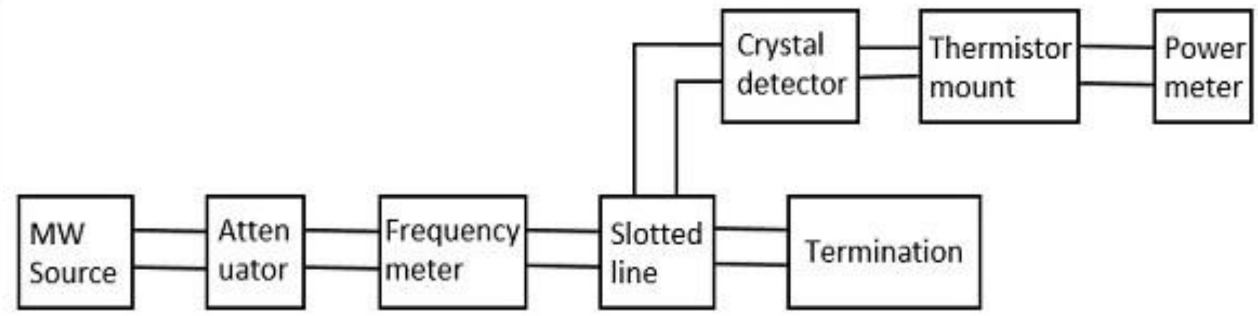
1. Power Ratio Method

In this method, the measurement of attenuation takes place in two steps.

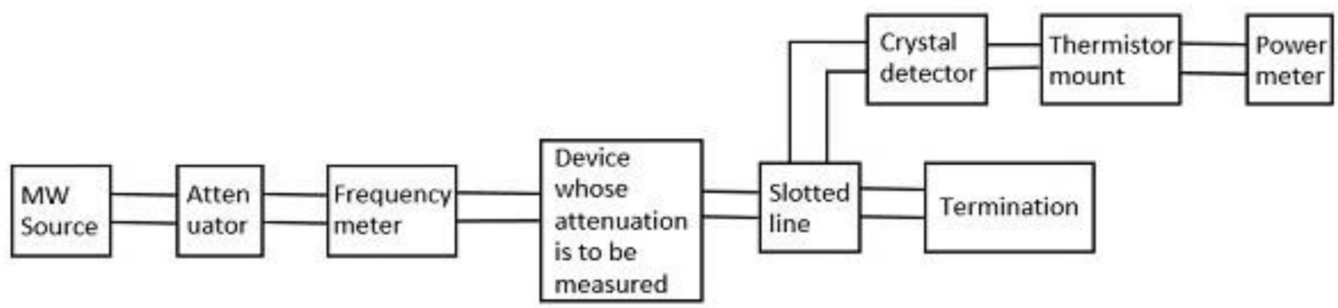
- **Step 1** − The input and output power of the whole Microwave bench is done without the device whose attenuation has to be calculated.
- **Step 2** − The input and output power of the whole Microwave bench is done with the device whose attenuation has to be calculated.

The ratio of these powers when compared, gives the value of attenuation.

The following figures are the two setups which explain this.



Set up 1, Power ratio method



Set up 2, Power ratio method

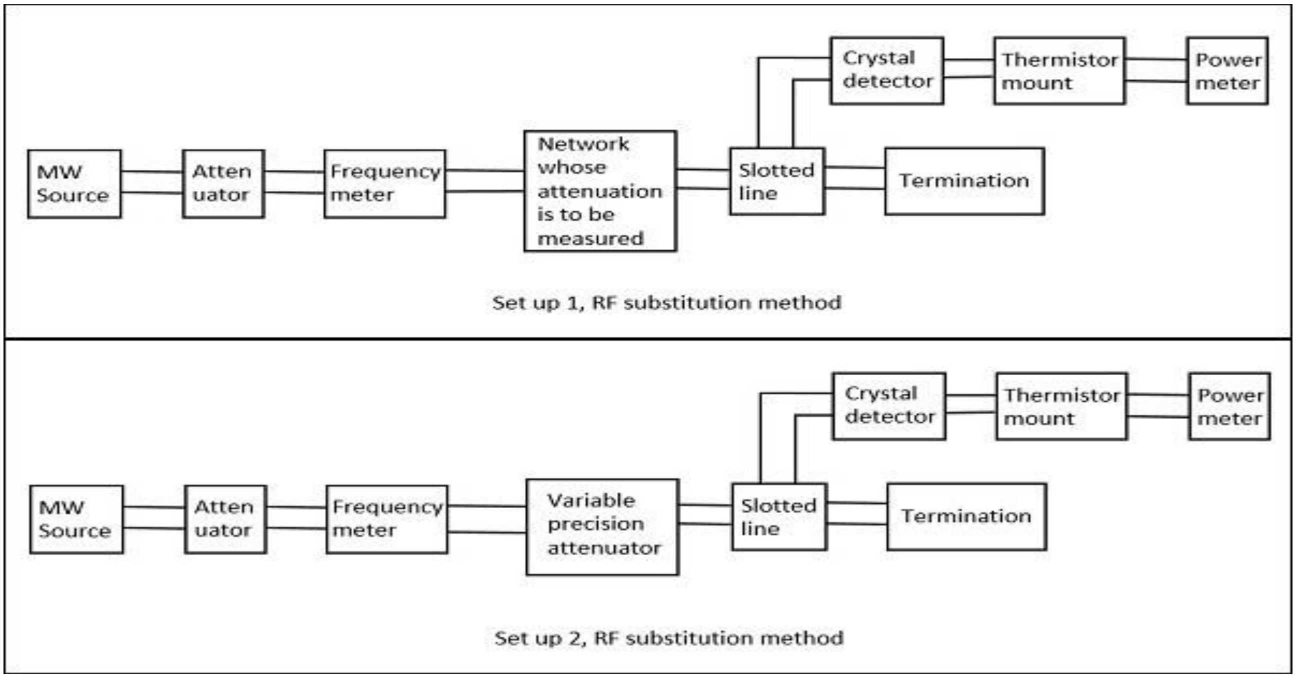
Drawback − the power and the attenuation measurements may not be accurate, when the input power is low and attenuation of the network is large.

2. RF Substitution Method

In this method, the measurement of attenuation takes place in three steps.

- **Step 1** − the output power of the whole Microwave bench is measured with the network whose attenuation has to be calculated.
- **Step 2** − The output power of the whole Microwave bench is measured by replacing the network with a precision calibrated attenuator.
- **Step 3** − Now, this attenuator is adjusted to obtain the same power as measured with the network.

The following figures are the two setups which explain this.



The adjusted value on the attenuator gives the attenuation of the network directly. The drawback in the above method is avoided here and hence this is a better procedure to measure the attenuation.

Frequency Measurement:

Mechanical techniques

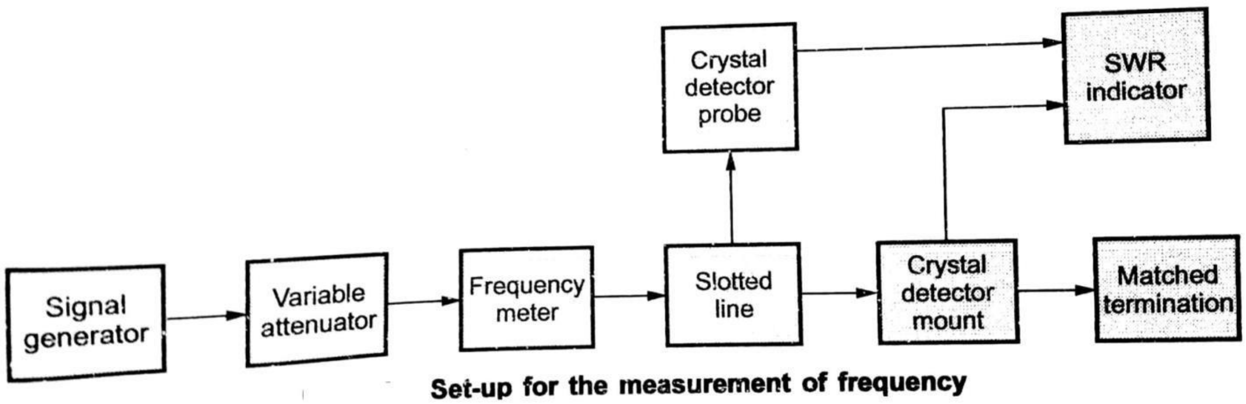
The mechanical techniques can be divided into two types

1. Slotted line technique
2. Cavity wave meter technique

The above techniques operation and accuracy depends upon the physical dimensions of the mechanical devices.

Slotted Line Technique

- A slotted line is a piece of transmission line and it is constructed in such a way that the voltage and current along it can be measured continuously over its length.
- The general set up for the measurement of microwave frequency is shown



Set-up for the measurement of frequency

- When a waveguide is mismatched by a load, a standing wave is created in the waveguide.
- The distance between the two adjacent maxima or minima is one half of the wavelength.
- Standing waves are set up in a slotted line producing minima every half wavelength apart.
- The distance between minima can be measured and guide wavelength hence frequency can be measured.

$$\frac{\lambda_g}{2} = (D_2 - D_1) = \Delta D$$

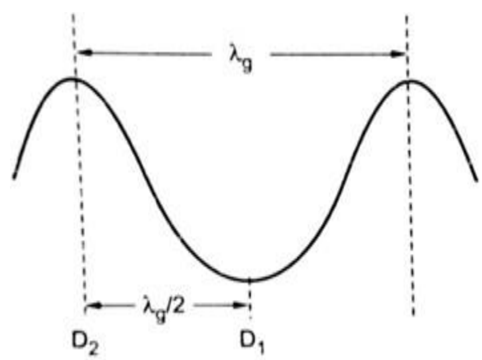
$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

But for TE_{10} mode

$$\lambda_c = 2a$$

and $$\lambda_0 = \frac{c}{f}$$

$$\therefore \quad f = \frac{c}{\lambda_0}$$

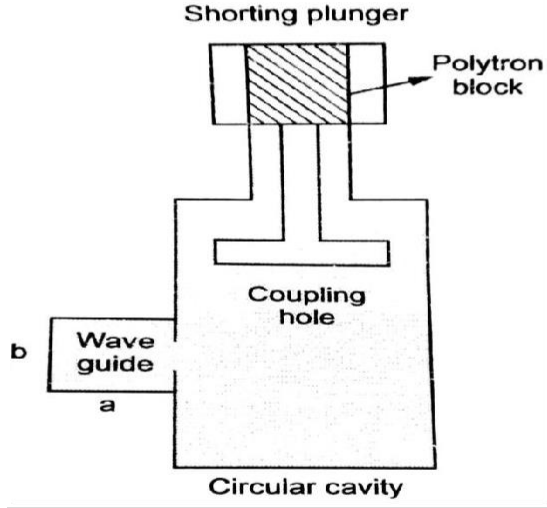


Maxima and minima of a wave

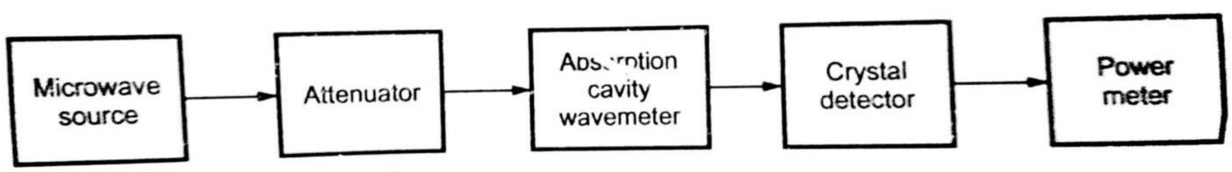
Cavity Wave meter Technique or Resonant Cavity Technique:

- A typical wave meter is a cylindrical cavity with a variable short circuit termination which changes the resonance frequency of the cavity by changing cavity length.
- Wave meter axis placed perpendicular to the broad wall of the waveguide.

- Wave meter axis is coupled by a hole in the narrow wall as shown
- A block of absorbing material placed at the back of the tuning plunger prevents oscillation on the top of it.
- Cavity resonates at different frequencies for different plunger positions.

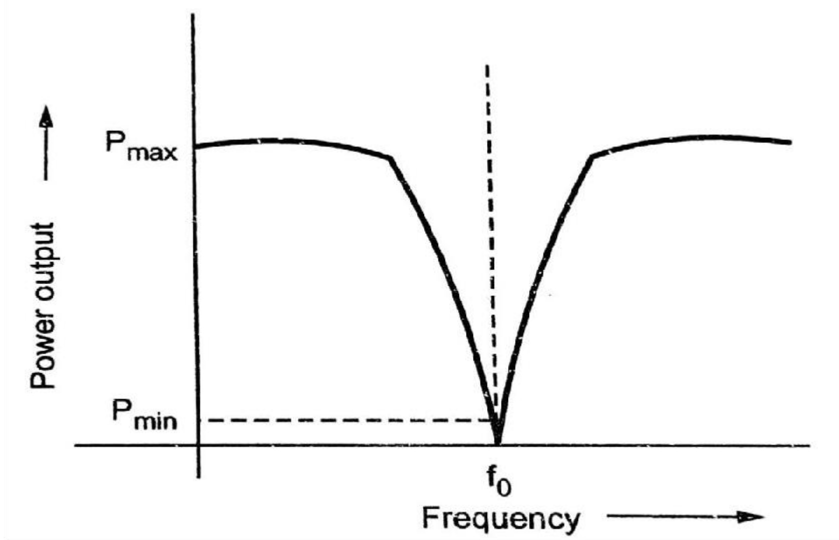


- The wave meter is called absorption type wave meter because the power is absorbed in wave meter at resonance and they attenuate the signal frequency to which they are tuned.
- The wave meter is called transitive cavity wave meter which passes the signal frequency to which they are tuned.
- The absorption type wave meters are preferred for the laboratory frequency measurement.
- The general set up for the frequency measurement by absorption type cavity wave meter is shown



- Microwave source is used to generate microwave signal.
- Attenuator is used to vary the microwave signal. Initially it is varied to get full scale reading P_{max}(take the reading from the power meter).

- Let the frequency of the microwave source (f_1)and the knob of wavemeter is set on frequency (f_i). Then the wavemeter is tuned to new frequency until the reading on the power meter dips to the minimum value (P_{min}).
- P_{min} value indicates that absorption cavity wavemeter is now at resonance and the new value of frequency read when this dip occurs will be the frequency f_2 of the microwave source.



Measurement of VSWR:

In any Microwave practical applications, any kind of impedance mismatches lead to the formation of standing waves. The strength of these standing waves ismeasured by Voltage Standing Wave Ratio the ratio of maximum to minimum voltage gives the VSWR which is denoted by S.

$$S = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho}$$

$$\text{where, } \rho = \text{ reflection coefficient} = \frac{P_{reflected}}{P_{incident}}$$

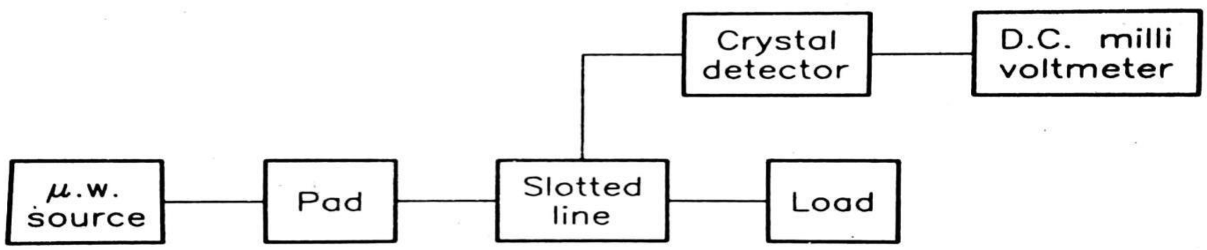
S varies from 1 to ∞

as ρ varies from 0 to 1

The measurement of VSWR can be done in two ways, Low VSWR and High VSWR measurements.

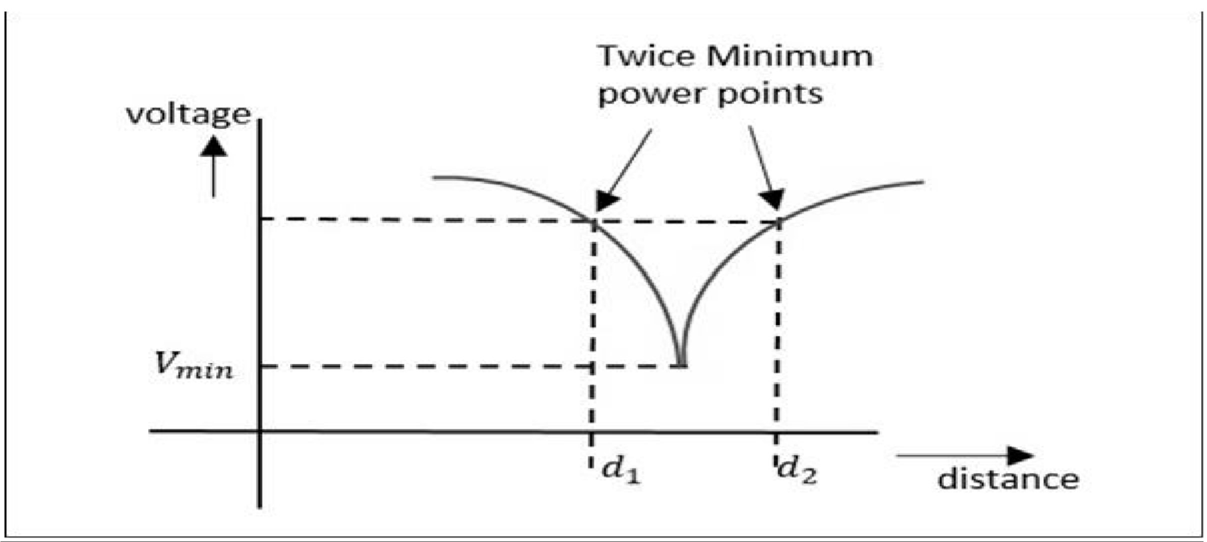
1. Measurement of Low VSWR S<10:

The measurement of low VSWR can be done by adjusting the attenuator to get a reading on a DC mill voltmeter which is VSWR meter. The readings can be taken by adjusting the slotted line and the attenuator in such a way that the DC mill voltmeter shows a full scale reading as well as a minimum reading.
Now these two readings are calculated to find out the VSWR of the network.



2. Measurement of High VSWR S>10:

The measurement of high VSWR whose value is greater than 10 can be measured by a method called the **double minimum method**. In this method, the reading at the minimum value is taken, and the readings at the half point of minimum value inthe crest before and the crest after are also taken. This can be understood by the following figure.



Now, the VSWR can be calculated by a relation, given as −

$$VSWR = \frac{\lambda_g}{\pi(d_2 - d_1)}$$

Where, λ_g *is the guided wavelength*

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (\frac{\lambda_0}{\lambda_c})^2}} \quad where\ \lambda_0 = c/f$$

As the two minimum points are being considered here, this is called as double minimum method.

Measurement of Impedance:

Impedance at microwave frequencies can be measured by using following 3 methods

- Using Magic T
- Using Slotted line
- Using Reflectometer

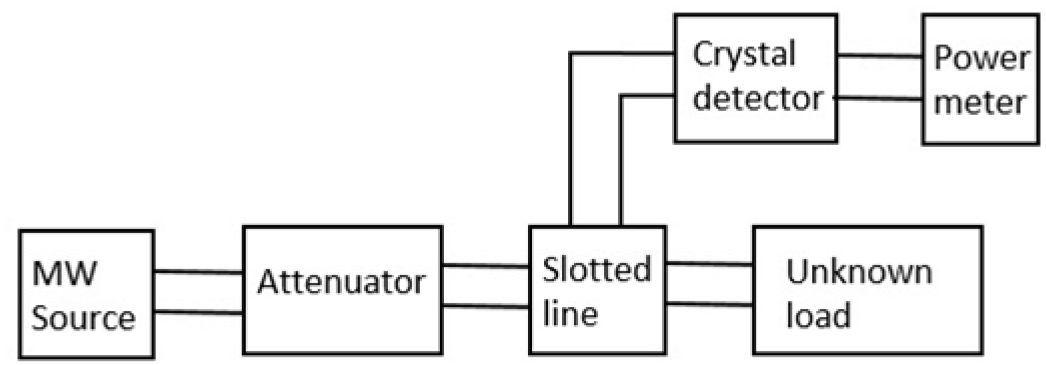
Apart from Magic Tee, we have two different methods, one is using the slotted line and the other is using the reflectometer.

1. Impedance Using the Slotted Line:

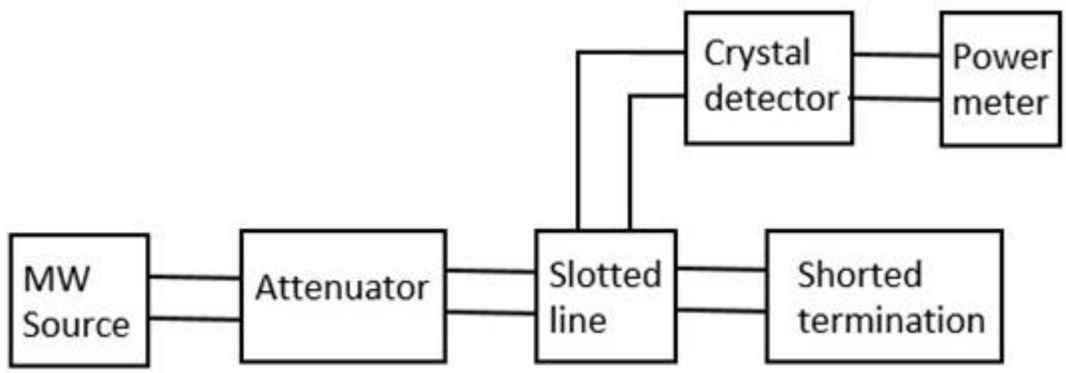
In this method, impedance is measured using slotted line and load ZL and by using this, Vmax and Vmin can be determined. In this method, the measurement of impedance takes place in two steps.

- **Step 1** − Determining Vmin using load ZL.
- **Step 2** − Determining Vmin by short circuiting the load.

This is shown in the following figures.

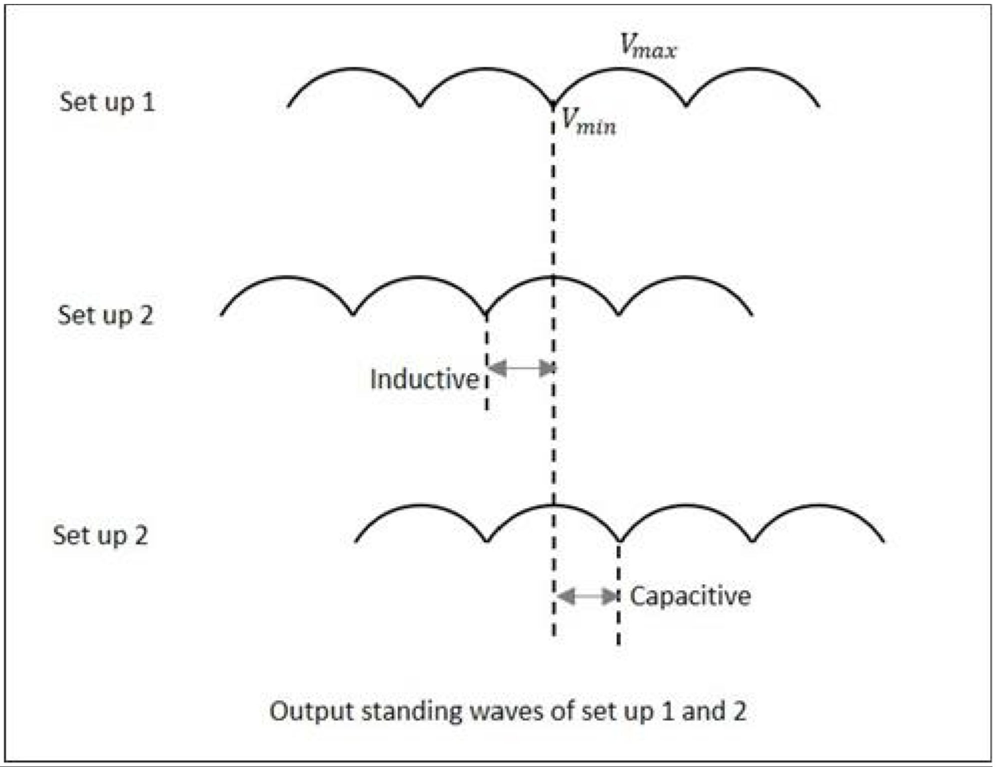


Set up 1, Impedance measurement using slotted line



Set up 2, Impedance measurement using slotted line

When we try to obtain the values of Vmax and Vmin using a load, we get certain values. However, if the same is done by short circuiting the load, the minimum gets shifted, either to the right or to the left. If this shift is to the left, it means that the load is inductive and if it the shift is to the right, it means that the load is capacitive in nature. The following figure explains this.

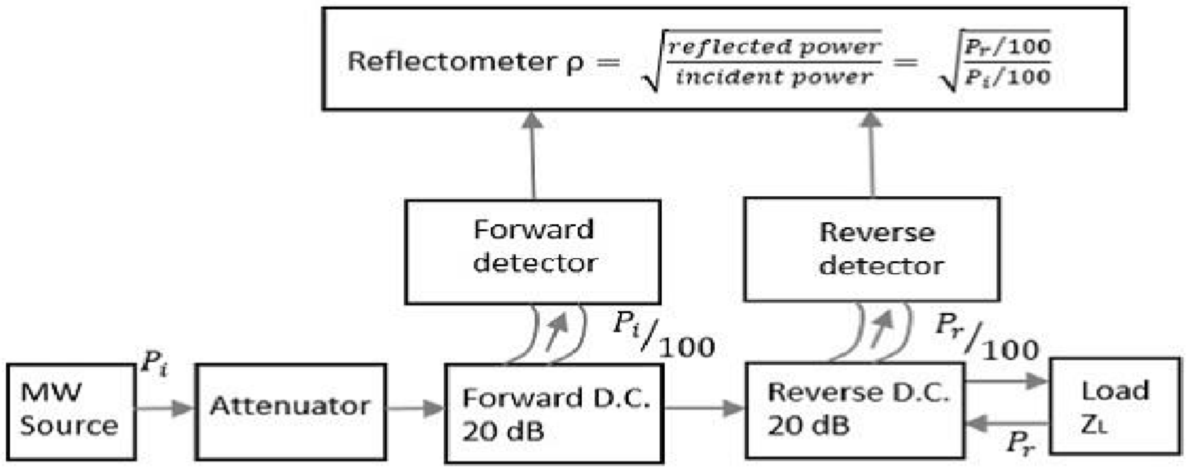


By recording the data, unknown impedance is calculated. The impedance and reflection coefficient ρ can be obtained in both magnitude and phase.

2. Impedance Using the Reflectometer:

Unlike slotted line, the Reflectometer helps to find only the magnitude of impedance and not the phase angle. In this method, two directional couplers which are identical but differs in direction are taken.

These two couplers are used in sampling the incident power Pi and reflected power Pr from the load. The reflectometer is connected as shown in the following figure. It is used to obtain the magnitude of reflection coefficient ρ, from which theimpedance can be obtained.



Set up for measuring impedance using reflectometer

From the reflectometer reading, we have

$$\rho = \sqrt{\frac{P_r}{P_i}}$$

From the value of ρ , the $VSWR$, i.e. S and the impedance can be calculated by

$$S = \frac{1+\rho}{1-\rho} \quad and \quad \frac{z - z_g}{z + z_g} = \rho$$

Where, zg is known wave impedance and z is unknown impedance.

Though the forward and reverse wave parameters are observed here, there will be no interference due to the directional property of the couplers. The attenuator helps in maintaining low input power.